The 30th International Conference on Difference Equations and Applications

ICDEA 2025

15 – 19 July 2025

Organized by

International Society of Difference Equations ISDE

ISDE

NSFC National Natural Science Foundation of China

广州大学 GUANGZHOU UNIVERSITY

A Letter from the Organizing Committee

Dear participants,

We welcome you to participate in the 30th International Conference on Difference Equations and Applications (ICDEA 2025), scheduled to take place from July 15 to 19, 2025, in Guangzhou, China. The conference will be held at the Ramada By Wyndham, located at the center of Guangzhou.

The annual ICDEA, organized by the International Society of Difference Equations (ISDE), is a prestigious event that brings together researchers and scientists from around the world. The conference aims to foster the presentation, discussion, and exploration of solutions in the fields of Difference Equations, Discrete Dynamical Systems, and their applications in various disciplines, including mathematical biology, epidemiology, evolutionary game theory, economics, physics, and engineering. ICDEA 2025 is organized by ISDE and Guangzhou University, funded by the National Natural Science Foundation of China (NSFC).

Furthermore, we are pleased to announce that we anticipate the active participation of over 300 attendees, in-person or online, at the conference. The talks will be presented through Regular Sessions and Special Sessions, covering a diverse range of research topics. Additionally, we have 13 esteemed Plenary Speakers who will delve into both theoretical aspects and practical applications in mathematical modeling.

We wholeheartedly look forward to your presence at ICDEA 2025 as we embark on a fruitful and enlightening conference.

Best regards,

Bo Zheng (Chair), Jianshe Yu (Co-chair)

A letter from the President of ISDE

Dear Colleagues,

It is my great pleasure to present the book of abstracts of the talks for the International Conference on Difference Equations and Applications (ICDEA 2025). On behalf of the International Society of Difference Equations (ISDE), I extend my warmest greetings to all the distinguished researchers, academics, and professionals who have contributed to this remarkable compilation of cutting-edge research.

This book showcases the depth and variety of scientific content in the talks at ICDEA 2025. The abstracts highlight diverse studies, analyses, and results from researchers exploring difference equations, discrete dynamical systems, and their applications in fields such as biology, economics, physics, and engineering.

The abstracts within this volume cover a wide range of topics, encapsulating the multifaceted nature of the conference. They encompass the fundamental aspects of difference equations and discrete dynamical systems, their mathematical properties, and the novel methodologies developed to analyze and understand their behavior.

I would like to extend my deepest gratitude to the contributors whose research is showcased in this book. Your dedication, expertise, and commitment to excellence have not only enriched this conference but have also advanced the collective understanding of difference equations and discrete dynamical systems. Your abstracts serve as a source of inspiration and a catalyst for further exploration in these dynamic fields.

I would also like to express my heartfelt appreciation to the members of the organizing committee for their tireless efforts in ensuring the success of the conference and the compilation of this book. Their diligence, attention to detail, and passion for knowledge have been instrumental in bringing together such a diverse and exceptional collection of abstracts.

As you peruse the pages of this book, I encourage you to embrace the spirit of collaboration, innovation, and intellectual curiosity that underpin the conference. Let these abstracts serve as catalysts for new ideas, fruitful discussions, and interdisciplinary collaborations. May they inspire researchers, educators, and practitioners alike to further explore the fascinating world of difference equations, discrete dynamical systems, and their applications to economics, science, and engineering.

Once again, I extend my warmest appreciation to all those who have contributed to this book of abstracts. Your work and dedication are vital in shaping the future of our society, and it is an honor to have you as part of our esteemed community.

With sincere gratitude and best wishes

Professor Laura Gardini

ISDE President

About ICDEA History

The previous conferences on Difference Equations and Applications were held in:

- 29th, Paris, France, June 24–28, 2024
- 28th, Phitsanulok, Thailand, July 17–21, 2023
- 27th, Paris-Saclay, France, July 18–22, 2022
- 26th, Sarajevo, Bosnia and Herzegovina, July 26–30, 2021
- 25th, London, UK, June 24–28, 2019
- 24th, Dresden, Germany, May 21–25, 2018
- 23rd, Timisoara, Romania, July 24–28, 2017
- 22nd, Osaka, Japan, July 24–29, 2016
- 21st, Bialystok, Poland, July 19–25, 2015
- 20th, Wuhan, Hubei, China, July 21–25, 2014
- 19th, Muscat, Oman, May 26–30, 2013
- 18th, Barcelona, Spain, July 24–29, 2012
- 17th, Trois-Rivières, Quebec, Canada, July 24–29, 2011
- 16th, Riga, Latvia, July 19–23, 2010
- 15th, Estoril, Portugal, October 19–23, 2009
- 14th, Istanbul, Turkey, July 21–25, 2008
- 12th, Lisbon, Portugal, July 23–27, 2007
- 11th, Kyoto, Japan, July 24–28, 2006
- 10th, Münich, Germany, July 25–30, 2005
- 9th, Los Angeles, California, USA, August 2–6, 2004
- 8th, Brno, Czech Republic, July 28–August 1, 2003
- 7th, Changsha, China, August 12–17, 2002
- 6th, Augsburg, Germany, July 20–August 3, 2001
- 5th, Temuco, Chile, January 2–7, 2000
- 4th, Poznan, Poland, August 27–31, 1998
- 3rd, Taipei, Taiwan, 1997
- 2nd, Veszprém, Hungary, 1995
- 1st, San Antonio, Texas, USA, 1994

Conference Committees

Local Organizing Committee from Guangzhou University

Bo Zheng (Chair) Jianshe Yu (Co-chair)

Dingyong Bai	Hongpeng Guo	Zhiming Guo	Linchao Hu	Feng Jiao	Chunming Ju
Jia Li	Yijie Li	Genghong Lin	Yunfeng Liu	Zhigang Liu	Yuhua Long
Qiwen Sun	Kai Wang	Huafeng Xiao	Hong Zhang	Hongling Zhou	Zhan Zhou

Scientific Committee

Chair

Jim Michael Cushing, University of Arizona, Tucson, Arizona

Members

- René Lozi, University Côte d'Azur, Nice, France
- Adina Luminita Sasu, West University of Timisoara, Timisoara, Romania
- Iryna Sushko, National Academy of Sciences of Ukraine, Kyiv, Ukraine
- Bo Zheng, Guangzhou University, Guangzhou, China

Sponsors

International Society of Difference Equations (ISDE)

Contents

A Letter from the Organizing Committee **ii**

A letter from the President of ISDE **iii**

About ICDEA History **iv**

Conference Committees **v**

Local Organizing Committee from Guangzhou University v

Scientific Committee v

Sponsors v

Plenary Speakers **1**

A journey into multiple attractors (Saber Elaydi) 1

Fingerprints' of wild chaos (Hinke Osinga) 3

Applied symbolic dynamics: from 1D to 2D to ODE (Wei-Mou Zheng) . . 4

From delay-independent to delay-dependent invariance: A journey through set-theoretic tools for time-delay systems (Sorin Olaru) 5

Iteration is eraser (Weinian Zhang) 6

Noise-controlled spatio-temporal structures in networks of chaotic maps (Galina Strelkova) 7

Diffeomorphisms with infinitely many Smale horseshoes (Xu Zhang) 9

What is the graph of a dynamical system? (James A. Yorke) 10

Trivial but interesting dynamics (Stephen Baigent) 11

Homoclinic chaos (B. Coomes, H. Kocak, and K. Palmer) 12

Discretizing delay systems can yield deep neural networks (Serhiy Yanchuk) 13

Fractional difference equations in machine learning (Guo-Cheng Wu) . . . 14

Discrete-time models for interactive dynamics of wild and sterile mosquitoes (Jianshe Yu) 15

Speaker **16**

Special Session 1 16

Special Session 2 22

Special Session 3 27

Special Session 4 32

Special Session 5 45

Special Session 6 54

Special Session 7 59

Special Session 8 69
Special Session 9 74
Special Session 10 84
Special Session 11 92
Special Session 12 96
Special Session 13 101
Special Session 14 107
Special Session 15 111
Special Session 16 116
Special Session 17 121
Special Session 18 124
Special Session 19 128
Special Session 20 134
Special Session 21 139
Special Session 22 143
Special Session 23 151
Special Session 24 156
Special Session 25 161
Special Session 26 166
Special Session 27 171
Special Session 28 176
Special Session 29 179
Special Session 30 183
Special Session 31 188
Special Session 32 192
Special Session 33 197
Special Session 34 203
Special Session 35 210
Special Session 36 215
Special Session 37 219
Special Session 38 224
Special Session 39 228
Regular 233

Plenary Speakers

A journey into multiple attractors

Saber Elaydi

Department of Mathematics, Trinity University, USA
E-mail: selaydi@trinity.edu

Abstract

Our main focus is on the introduction and analysis of discrete-time population models that exhibit multiple attractors. Most widely studied single-species density-dependent models without structure - such as the Beverton-Holt equation, the Leslie-Gower competition model, and the Ricker competition model-are generally incapable of supporting multiple attractors. However, as demonstrated in [1], even the Leslie-Gower model can admit multiple boundary attractors when interior equilibria are absent. In this talk, we extend our investigation to include a wide variety of structured and unstructured models - single-species and multi-species alike-that support multistability.

When multiple attractors occur, the long-term fate of a population becomes strongly dependent on initial conditions. This implies that one cannot predict all possible outcomes based on a single trajectory or data set. The phenomenon of deterministic uncertainty is exacerbated in the presence of environmental noise, which can induce transitions between basins of attraction. When coexisting attractors include cycles or chaotic sets, and the system exhibits sensitive dependence on initial conditions, the basins of attraction may become intricately intermingled, further complicating prediction [4].

A particularly well-known mechanism leading to multiple attractors is the Allee effect, where the per capita growth rate increases at low population densities due to biological advantages of aggregation, such as enhanced mating success, group

foraging, or protection from predators [2, 5, 6]. A strong Allee effect occurs when the per capita growth rate fails to exceed one until the population crosses a critical threshold, resulting in two attractors: extinction and a survival state. These effects arise from positive density dependence in components of fitness (e.g., birth rate or survival rate), in contrast to the negative density dependence found in classical models like Beverton-Holt or Ricker.

We examine both unstructured and structured models exhibiting multistability through various mechanisms, including strong Allee effects, double backward bifurcations, and hysteresis loops in bifurcation diagrams [6]. Examples include semelparous Leslie models where juvenile survival increases with density, yielding three attractors: extinction, a stable positive equilibrium, and a stable 2-cycle. We also study two-patch dispersal models that display bistability in the absence of Allee effects, as well as multi-species competition models, predator-prey models, age-structured models, and epidemic systems. These models illustrate how biologically motivated nonlinearities and structure can yield rich dynamical outcomes. Our treatment follows and expands the approaches of [1] and [3], combining bifurcation analysis with biological interpretation.

References

[1] J. C. Alexander, J. A. Yorke, Z. You, and I. Kan, "Riddled basins," International Journal of Bifurcation and Chaos, 2(4):795-813, 1992.

[2] F. Courchamp, L. Berec, and J. Gascoigne, Allee Effects in Ecology and Conservation, Oxford University Press, 2008.

[3] J. M. Cushing, An Introduction to Structured Population Dynamics, SIAM, 1998.

[4] S. Elaydi and R. Sacker, Population models with Allee effects: A new model,' Journal of Biological Dynamics, 4(4):397-408, 2010.

[5] S. Elaydi and J. M. Cushing, Discrete Mathematical Models in Population Biology: Ecological, Epidemic, and Evolutionary Dynamics, Springer, 2025.

[6] S. Elaydi, E. Kwessi, G. Livadiotis, Hierarchical competition models with the Allee effect III: multispecies, Journal of Biological Dynamics, 12(1): 271-287, 2018.

Fingerprints' of wild chaos

Hinke Osinga

Department of Mathematics, The University of Auckland, New Zealand
E-mail: h.m.osinga@auckland.ac.nz

Abstract

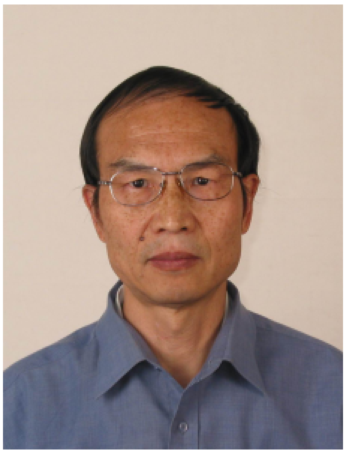
In a smooth world, curves, surfaces, and more generally, manifolds intersect each other transversely, if at all; tangencies are rare events that cannot be observed in experiments. In dynamical systems theory, therefore, it was long assumed that tangencies between invariant manifolds occur at isolated points when a parameter is varied, and the transition from tame to chaotic dynamics is mediated by a single tangential event. Recent theoretical work by Bonatti, Diaz, and others has shown that the boundary between tame and chaotic dynamics is, actually, more like a thick grey world that challenges our geometric intuition: tangencies may occur robustly, which is called wild chaos. This type of dynamics requires at least three dimensions for discrete-time systems, or four for a system of ordinary differential equations. This higher dimensionality has been an impediment to our understanding of how steady states, periodic solutions, and their invariant manifolds organise wild chaotic dynamics.

In this talk, I will discuss how manifolds can have persistent tangential intersections as a parameter is varied. The starting point will be classical chaos in the planar Hénon map, a simple polynomial dynamical system. I will then consider a three-dimensional extension and explain its counter-intuitive properties: this map has one-dimensional invariant manifolds that cannot be avoided by other smooth curves. Hence, these one-dimensional manifolds behave as though they are two dimensional. With a careful combination of dynamical systems theory and advanced computational methods, I will show what wild chaos and robust tangencies look like, how they arise, and why this matters for applications.

Applied symbolic dynamics: from 1D to 2D to ODE

WEI-MOU ZHENG

Institute of Theoretical Physics Chinese Academy of Sciences, China
E-mail: zheng@mail.itp.ac.cn



Abstract

Symbolic dynamics provides almost the only rigorous way to understand global systematics of periodic and, especially, chaotic motion in dynamical systems. The method of applied symbolic dynamics has been developed to such an extent that it may well become a practical tool in studying chaotic dynamics, especially for 1D maps. Starting with the unimodal map on the interval, some fundamental concepts such as the partition of the phase space according to the monotonicity of the mapping function, the ordering of symbolic sequences and conditions for admissible sequences are explained. The extension of symbolic dynamics from 1D to 2D maps is a great challenge simply because there is no simple ordering for points on the 2D map. With the help of the Lozi map, whose partition of the phase space is given, a prototype of 2D symbolic dynamics is defined. The Hénon map is then studied without constructing dynamical foliations by assuming that it belongs to the same prototype of symbolic dynamics as the Lozi map. By means of the Poincaré surface of section, the flow of the Lorenz equations is converted to a 2D map, and the application of the symbolic dynamics to the Lorenz equations is briefly discussed.

From delay-independent to delay-dependent invariance: A journey through set-theoretic tools for time-delay systems

Sorin Olaru

CentraleSupélec, Paris Saclay University, France
E-mail: sorin.olaru@centralesupelec.fr

Abstract

Time-delay systems are omnipresent in control applications where transmission, actuation, or sensing delays play a critical role. Understanding their behavior through the lens of positive invariance opens the door to robustness and constraint satisfaction guarantees in control design. This talk bridges classical delay-independent invariance concepts with emerging delay-dependent formulations.

Starting from the foundational notion of D-invariance, we explore the set-theoretic characterization of invariant sets for linear discrete-time delay systems. Particular attention is given to how D-invariance offers a delay-independent perspective with fixed prescribed complexity, yet lacking full characterizability.

Building on this, we introduce recent developments, which incorporate delay-dependent invariance notions. These offer refined tools by adapting the invariant set definition to account for specific delay values and their variability. This shift enables improved conservativeness in controller synthesis and stability analysis, especially when dealing with time-varying or uncertain delays.

The talk will illustrate the theoretical underpinnings, necessary and sufficient conditions, and set factorization approaches, along with practical algorithms and examples. We will emphasize how the combination of geometric, algebraic, and iterative tools leads to a more nuanced understanding of system invariance in the presence of delays.

Iteration is eraser

WEINIAN ZHANG

Sichuan University, China
E-mail: matzwn@126.com, wnzhang@scu.edu.cn

Abstract

Usually, the complexity of dynamics is understood as a result of iterated nonlinearity. In this talk we show that some "tumors"–bad terms or bad points can be erased by iteration. We introduce results on degree-preserving and repair of continuity and smoothness under iteration.

Noise-controlled spatio-temporal structures in networks of chaotic maps

Galina Strelkova

Institute of Physics, Saratov State University, Saratov, Russia
E-mail: strelkovagi@sgu.ru

Abstract

The present lecture devotes to the review of recently obtained numerical results concerning the impact of noise on the dynamics and spatio-temporal patterns in networks of discrete-time nonlinear systems. As individual elements three well-known chaotic maps are chosen, the Hénon map, the Lozi map, and the Hénon-Lozi map. The ring network of nonlocally coupled Hénon maps is characterized by the appearance of chimera states which accompany the transition from complete synchronization to spatio-temporal chaos with decreasing the coupling strength between nodes [4]. The chimera state corresponds to the coexistence of clusters with coherent and incoherent dynamics of elements of a network [2]. The ring network of nonlocally coupled Lozi maps demonstrates another scenario of the transition, which is related to the appearance of solitary states [3]. Solitary states denote network regimes when solitary nodes coexist with coherent or synchronized groups of elements [8]. The Hénon-Lozi map represents a synthesis of the Hénon and Lozi maps and combines their dynamic properties [1]. Thanking this feature, the ring network of coupled Hénon-Lozi maps can exhibit both chimera states and solitary states.

It is shown that chimeras and solitary states demonstrate different response and robustness toward external noise perturbations. The dynamics of three aforementioned networks of coupled maps are systematically studied in the presence of noise, including Lévy noise, when the coupling parameters of the networks and the noise characteristics are varied. It is shown that additive and multiplicative noise can increase the probability of observing chimera states in ring networks of of nonlocally

coupled Hénon maps and Hénon-Lozi maps [5–7]. This means that there is an optimum noise level at which the interval of the coupling strength within which chimeras are observed with a high or even maximum probability is the widest. This effect constitutes the constructive role of noise in analogy with stochastic and coherence resonance and is referred to as chimera resonance [5].

It is revealed that even weak noise can sustain the solitary state regime in the range of weak nonlocal coupling in the networks of nonlocally coupled Lozi maps and Hénon-Lozi maps. However, stronger noise causes the solitary nodes to disappear for any values of the coupling strength [7].

Thus, the obtained results clearly indicate that external noise can serve as an effective tool for controlling the spatio-temporal dynamics and structures of complex networks.

References

[1] M. Aziz-Alaoui, C. Robert, C. Grebogi, Dynamics of a Hénon-Lozi-type map. Chaos, Solitons and Fractals 12(12), 2323 (2001).

[2] Y. Kuramoto and D. Battogtokh, Nonlinear Phenom. Complex Syst. 5, 380 (2002).

[3] Y. Maistrenko, B. Penkovsky, and M. Rosenblum, Phys. Rev. E 89, 060901 (2014).

[4] I. Omelchenko, B. Riemenschneider, P. Hövel, Y. Maistrenko, E. Schöll, Phys. Rev. E 85(2), 026212 (2012).

[5] E. Rybalova, V. Nechaev, E. Schöll, and G. Strelkova, Chaos 33, 093138 (2023).

[6] E. Rybalova, E. Schöll, and G. Strelkova, J. Diff. Equ. Appl. 29(9-12), 909 (2023).

[7] E.V. Rybalova, V.M. Averyanov, R. Lozi, G.I. Strelkova, Chaos, Solitons and Fractals 184, 115051 (2024).

[8] N. Semenova, A. Zakharova, E. Schöll, V. Anishchenko, Europhys. Lett. 112(4), 40002 (2015).

Diffeomorphisms with infinitely many Smale horseshoes

Xu Zhang

Shandong University, Weihai, China
E-mail: xu_zhang_sdu@mail.sdu.edu.cn

Abstract

The well-known Smale horseshoe is the topological symbol of the existence of chaotic dynamics. In this talk, a class of planar diffeomorphims will be introduced, with infinitely many coexisting Smale horseshoes, where the Lebesgue measure of the parameters with such strange dynamics is infinite. On each horseshoe, there exists a uniformly hyperbolic invariant set, on which the map is topologically conjugate to the two-sided full-shift on two symbols. Moreover, the topological entropy is infinite in certain parameter regions. Some extensions will also be included.

What is the graph of a dynamical system?

James A. Yorke

Institute for Physical Science and Technology, University of Maryland, USA
E-mail: yorke@umd.edu

Abstract

Frequently we describe a dynamical system in terms of an attractor. That can leave out important information about unstable invariant sets. These represent hidden behavior that can become important when the system is perturbed. The graph of a dynamical system enables us to try to encapsulate some of the complexity. Some examples of why this is important will be presented, from the very simple to the very complex. I will emphasize the logistic map, with which Roberto De Leo began this project.

References

[1] Roberto De Leo and James A. Yorke, The graph of the logistic map is a tower, Discrete and Continuous Dynamical Systems, 2021, 41(11): 5243-5269.

[2] Roberto De Leo and James A. Yorke, Infinite towers in the graphs of many dynamical systems, Nonlinear Dynamics, 2021, 105(1), pp.813-835.

Trivial but interesting dynamics

STEPHEN BAIGENT

Department of Mathematics, University College of London, UK
E-mail: steve.baigent@ucl.ac.uk

Abstract

A map is said to exhibit trivial dynamics when all omega limit sets are subsets of fixed points. We review some trivial dynamics in some ecological maps by way of topological methods, monotone systems theory and Lyapunov functions. Then we look specifically at the case where the map is planar and non-injective, such as for the Ricker map, and obtain interesting results on stable and unstable sets that can be viewed as a version of the competitive exclusion principle that is well-known in ecology.

Homoclinic chaos

B. Coomes, H. Kocak and K. Palmer

Institute for Physical Science and Technology, University of Maryland, USA
E-mail: coomes@math.miami.edu



Abstract

As first realized by Poincaré during his investigations of the restricted three-body problem, the presence of transversal homoclinic points (transversal intersections of stable and unstable manifolds of hyperbolic fixed points or periodic orbits) imply the existence of complicated, chaotic, dynamics. Rigorously proving the existence of such points in specific systems, however, turns out to be a challenging task. In this talk, we first present a general theorem for proving the existence of transversal homoclinic points near numerically computed approximate ones in diffeomorphisms. We then apply our theorem to establish the existence of transversal homoclinic points, hence the presence of chaotic dynamics, in several specific systems, including a perennial grass model, the Duffing map, and the Bogdanov map.

Discretizing delay systems can yield deep neural networks

Serhiy Yanchuk

School of Mathematical Sciences, University College Cork, Ireland
E-mail: syanchuk@ucc.ie

Abstract

In this presentation, I will review our results [1, 2], which demonstrate that a high-dimensional map in the form of a deep neural network (DNN) can be modelled using a single delay-differential equation with multiple delays. This single-neuron DNN comprises only a single nonlinearity and appropriately adjusted modulations of the feedback signals. Network states emerge over time as a temporal unfolding of the neuron's dynamics. By adjusting the feedback modulation within the loops, we adapt the network's connection weights. These connection weights are determined via a back-propagation algorithm, in which both the delay-induced and local network connections must be taken into account. Our approach can fully represent standard DNN and extends the DNN concept toward dynamical systems implementations. This new method is called Folded-in-Time DNN (Fit-DNN), and has been tested in a set of benchmark tasks.

References

[1] Florian Stelzer, André Röhm, Raul Vicente, Ingo Fischer, Serhiy Yanchuk, Deep neural networks using a single neuron: folded-in-time architecture using feedback-modulated delay loops. Nat Commun 12, 5164 (2021).

[2] Stelzer, Yanchuk, Emulating complex networks with a single delay differential equation. Eur. Phys. J. Spec. Top. 230, 2865–2874 (2021).

Fractional difference equations in machine learning

Guo-Cheng Wu

Key Laboratory of Intelligent Analysis and Decision on Complex Systems,
Chongqing University of Posts and Telecommunications, China
E-mail: wuguocheng@gmail.com

Abstract

This talks suggests discrete fractional calculus for numerical discretization of fractional differential equations. First, the physical meaning and background of fractional calculus are revisited. The traditional numerical methods face challenges in initial value problems. Then, a fractional difference equation approach is employed to address the initial value non-smoothness's problem. The convergence result is provided. Finally, applications in data-driven fractional differential equations and machine learning are considered.

Discrete-time models for interactive dynamics of wild and sterile mosquitoes

Jianshe Yu

Guangzhou Center for Applied Mathematics, Guangzhou University, China
E-mail: jsyu@gzhu.edu.cn

Abstract

To study the dynamics of interactive wild and sterile mosquitoes and their impacts on the transmissions of mosquito-borne diseases, various mathematical models have been formulated and investigated. In this talk, we present several discrete-time models based on difference equations. The survival functions are assumed to be of either Ricker-type or Beverton-Holt-type nonlinearity. The time steps for the homogeneous and stage-structured models are in more general settings, and births and survivals are included at each time step. The vital parameters are either constant or periodic. We study the interactive dynamics of wild and sterile mosquitoes where only sexually active sterile mosquitoes are considered. We determine threshold values of releases of sterile mosquitoes and investigate the model dynamics for both homogeneous and stage-structured populations. We also discuss the models where sterile mosquitoes are released periodically and impulsively. Some open questions are proposed for further studies.

Speaker

Special Session 1

Resonant grazing bifurcations in simple impacting systems

Indranil Ghosh*†[1] and David Simpson[1]

[1] *Massey University, New Zealand*
† *E-mail: i.ghosh@massey.ac.nz*

Abstract

Many practical engineering systems include vibrations and impacts. Recurring impacts are onset by grazing bifurcations, and these generate complex dynamics including chaos because the corresponding Poincaré map has a highly nonlinear and destabilising squareroot term. The main goal of this talk is to determine why leading-order approximations to such maps, such as the Nordmark map, are often only effective in extremely small parameter ranges. We find that this can be caused by a resonance effect, resulting in nearby period-doubling and saddle-node bifurcations. To numerically continue curves of these bifurcations, we found it helpful to develop a new numerical technique that allows us to use root-finding methods such as Newton's method without the method failing by effectively falling off theside of the square root.

Exploring blenders of a Hénon-like map in the presence of periodic or chaotic attractors

Sanaz Amani*†1

[1] *University of Auckland*
† *E-mail: Sama039@aucklanduni.ac.nz*

Abstract

A blender is a compact hyperbolic set of a diffeomorphism of dimension at least three, characterised by the feature that at least one of its invariant manifolds behaves as a geometric object of higher dimension. We investigate the existence of a blender in a three-dimensional Hénon-like map through a numerical test that checks what we call the carpet property. Specifically, we compute the one-dimensional manifold of a fixed or periodic point contained in the hyperbolic set, up to a very long arclength; we then investigate how increasingly longer pieces of manifold are weaving back and forth to form an object that is dense when viewed in a suitable projection. The Hénon-like map with a full horseshoe can have a blender exhibit the carpet property. Here, we study the regime where this map a chaotic attractor interspersed by periodic windows. We demonstrate that, perhaps surprisingly, the carpet property is satisfied throughout this regime.

This is joint work with Hinke M. Osinga and Bernd Krauskopf.

On the creation of a robust heterodimensional cycle near a homoclinic tangency

Katsutoshi Shinohara*†1

[1] *Hitotsubashi University*
† *E-mail: ka.shinohara@r.hit-u.ac.jp*

Abstract

We consider dynamical systems generated by the iteration of diffeomorphisms on smooth manifolds. Recently, research on non-uniformly hyperbolic systems has attracted attention. Homoclinic tangencies (HTs) and heterodimensional cycles (HDCs) are two key mechanisms in the formation of robust non-hyperbolic dynamical systems. Although these mechanisms are conceptually distinct, in some cases, one can give rise to the other. In this talk, I will review recent progress in understanding the formation of robust HDCs near a special type of HT.

This work is a collaboration with D. Li, D. Turaev (Imperial College London), and X. Li (Huazhong University of Science and Technology).

References

[1] Robust heterodimensional cycles in two-parameter unfolding of homoclinic tangencies, https://arxiv.org/abs/2203.14075

Interactions of invariant manifolds of a wildly chaotic hyperbolic set

Sam Doak[*†1]

[1] *University of Auckland*
[†] *E-mail: sdoa857@aucklanduni.ac.nz*

Abstract

Wild chaos is a type of robust higher-dimensional chaotic dynamics. One of its defining characteristics is the presence of robust heterodimensional cycles with non-transverse intersections between stable and unstable manifolds. This is counter-intuitive: for example, in three dimensions, it implies robust intersections between two one-dimensional manifolds. Consequently, at least one such curve must 'behave' as if it were a surface.

There are very few explicit examples of a discrete-time dynamical system exhibiting wild chaos. We study a three-dimensional quadratic diffeomorphism that is volume preserving. We illustrate the emergence of wild chaos in this system by first considering the one-dimensional manifolds of its two fixed points. Each fixed point lies in a different hyperbolic set, distinguished by the dimensions of their unstable manifolds. The one-dimensional manifolds of these two hyperbolic sets have 'surface-like' properties and intersect to give rise to robust heterodimensional cycles. We also discuss how these manifolds disentangle from each other as a parameter is varied, thereby highlighting specific changes in the manifold geometry that represent a mechanism to produce wild chaos.

This is joint work with Hinke M. Osinga and Bernd Krauskopf.

An abundance of heterodimensional cycles via period doubling

BERND KRAUSKOPF*†1

[1] *University of Auckland*
† *E-mail: b.krauskopf@auckland.ac.nz*

Abstract

A heterodimensional cycle consists of a pair of heteroclinic connections between two saddle periodic orbits with unstable manifolds of different dimensions. While any heterodimensional cycle is structurally unstable, amazingly, their existence is a robust phenomenon (in the C^1-topology) — showing that structural instability is not exceptional in phase spaces of sufficient dimension.

We show how heterodimensional cycles arise in a four-dimensional vector field (the minimal dimension in this context) and the associated three-dimensional Poincaré map. More specifically, we start by identifying a 'basic' heterodimensional cycle and then continue it in two system parameters until one of the saddle periodic orbits undergoes a period-doubling bifurcation. We show how this bifurcation generates new families of heterodimensional cycles with different geometric properties. In fact, this proccess repeats and we find an abundance of heterodimensional cycles of different types that is created via a cascade of period-doubling bifurcations.

This is joint work with Nelson Wong and Hinke Osinga.

Threshold mechanisms in p53 dynamics triggered by DNA damage

Yancong Xu*†1

[1] *China Jiliang University, Hangzhou, China*
† *E-mail: yancongx@cjlu.edu.cn*

Abstract

Threshold mechanisms have attracted a considerable attention in the process of DNA damage repair. In this talk, we will discuss the existence of isolas of equilibria and limit cycles in p53-Mdm2 network models using the dynamical system approach. By coincidence, both of them indicate two new threshold mechanisms for non-oscillatory and oscillatory DNA damage repair, and then determine cell cycle arrest and apoptosis. Of particular interest is that multiple oscillations and the threshold value of DNA damage are found to reveal the intrinsic mechanism of DNA damage repair.

Special Session 2

A stage-structured continuous-/discrete-time population model: Persistence and spatial spread

Minglong Wang[1], Yuxiang Zhang[2] and Qihua Huang*†[1]

[1]*Southwest University, China*
[2]*Tianjin University of Technology and Education, China*
†*E-mail: qihua@swu.edu.cn*

Abstract

Population persistence and spatial propagation and their dependence on demography and dispersal are of great importance in spatial ecology. Many species with highly structured life cycles invade new habitats through the dispersal of organisms in their early life stages (e.g., seeds, larvae, etc.). We develop a stage-structured continuous/discrete-time hybrid model to describe the spatiotemporal dynamics of such species, in which a reaction-diffusion equation describes the random movement of dispersing individuals, while two difference equations describe the demography of sedentary individuals. We obtain a formula for the spreading speed of the population in terms of model parameters. We show that the spreading speed can be characterized as the slowest wave speed of a class of traveling wave solutions. We provide an explicit formula for the critical domain size that separates population persistence from extinction. By comparing our stage-structured model with a physically unstructured model, we find that the structured model reduces to the unstructured one in some special cases. Accordingly, the results about the spreading speed and the critical domain size for the unstructured model represent some special cases of those for the structured one. This highlights the significance of including stage structure in studying the spatial dynamics of species with complex life cycles.

Stability of stochastic discrete epidemic models with general nonlinear incidence rate

MAOXING LIU*†1

1 *Beijing University of Civil Engineering and Architecture*
† *E-mail: liumaoxing@bucea.edu.cn*

Abstract

The purpose of this talk is to study the stability of stochastic discrete epidemic models with a general nonlinear incidence rate at the equilibrium. Based on a continuous SIS model with stochastic white noise directly proportional to the deviation of the state from the equilibrium, the model is discretized by the Euler-Marryama method, and then the stochastic discrete SIS epidemic model is obtained. A sufficient condition for the stability in probability of the nonlinear difference equations with stochastic perturbations at the equilibriums is proposed. The sufficient conditions for the stochastic stability of the model at positive equilibrium and boundary equilibrium are obtained. Numerical simulations show the influence of noise intensity on the stability of stochastic discrete model.

References

[1] X. Mao, Stochastic Differential Equations and Applications, 2nd ed., Academic Press, Cambridge, 2006.

[2] X. Mao, G.G. Yin, and C. Yuan, Stabilization and destabilization of hybrid systems of stochastic differential equations, Automatica 43 (2007), pp. 264-273.

[3] L. Shaikhet, Lyapunov Functionals and Stability of Stochastic Difference Equations, Springer, London, 2011.

[4] M. Liu, R. Yuan, Stability of a stochastic discrete SIS epidemic model with general nonlinear incidence rate, Journal of Difference Equations and Applications, 28(4):561-577, 2022.

Bifurcation and turing instability analysis for a space-time discrete carbon emission-absorption system

Maksude Keleş Denk*†[1,2] and Canan Çelik[2]

[1] *Nisantası University, Istanbul, Turkey*
[2] *Yildiz Technical University (YTU), Istanbul, Turkey*
† *E-mail: maksude.keles@nisantasi.edu.tr*

Abstract

In this work, the complex dynamics behavior of a space- and time-discrete carbon emission and carbon absorption model is studied. The discrete model is given by coupled map lattices (CMLs), taking a nonlinear relationship between carbon emission-absorption reaction stage and dispersal stage. Through the stability analysis, the conditions are obtained to ensure the stability of the homogeneous steady state of the system. Based on bifurcation theory, as well as the center manifold theorem, we analytically obtained the existence of flip bifurcation at the system's positive fixed point. Then, we derive the conditions for producing Turing instability of the discrete system. Finally, numerical simulations have been performed to verify and extend the theoretical results. In order to distinguish chaos from regular behaviors, the maximum Lyapunov exponents are shown. These findings demonstrate complex dynamics behaviors such as period-doubling cascade, chaotic behaviors and pattern formations.

Keywords: Flip Bifurcation, Chaos, Turing instability, Discrete model, Coupled map lattices

Global stability and Hopf bifurcation in a delay mosquito population suppression model with a constant release policy

YUFENG WANG*†1

[1] *School of Mathematics, South China University of Technology, China*
† *E-mail: yufengwang0930@163.com*

Abstract

We develop and analyze a mosquito population suppression delay model that incorporates survival probability during the maturation process. The model allows the trivial equilibrium to coexist with two positive equilibria and exhibits steady-state behavior including asymptotic stability, semi-stability and bistability. We analyze the subsets of basins of attraction for each stable equilibrium. Using delay as a bifurcation parameter, we examine the onset and global continuation of Hopf bifurcating periodic solutions by the global Hopf bifurcation theorem and the Bendixson criterion. Finally, some numerical examples are provided to validate the theoretical findings. This is a joint work with Rong Yan, Jianshe Yu and Jianhong Wu.

Keywords: Asymptotic stability, Global Hopf bifurcation

Non standard finite difference in SI epidemiological model: Bifurcation and control

Nurul Huda Gazi*†1

1 *Aliah University, IIA/27, Newtown, Kolkata-700014, INDIA, India*
† *E-mail: nursha12@gmail.com*

Abstract

In general, a discrete-type dynamical system exhibits richer dynamical phenomena than that of its continuous counterpart. Keeping this in mind, an epidemiological system is considered with two species: the susceptible and the infective. There is a constant recruitment of the susceptible in the system. The rate of infection is proportional to the contact rate of the infective species with the susceptible species. The susceptible population has a natural death rate and the infective population also decreases due to the natural death rate as well as the death due to disease. There is a treatment program for the infected population. The rate of treatment increases with the increase of the infected population [1]. The model is transformed into a discrete system by using NSFD method [2]. The solution is found to be positive. There are some conditions for the existence and feasibility of the fixed points. The stability property is analyzed using variational methods for the map in the neighborhood of the fixed points [3, 4]. The 2-dimensional map has been transformed into a feedback control system. Then it is analyzed to control its dynamical behaviour. Numerical computation is carried out to verify the analytical findings.

Keywords: SI model, NSFD, Positivity, Fixed point, Stability, Bifurcation, Control

References

[1] S. A. Gazi, N. H. The Effect of the Force of Infection and Treatment on the Disease Dynamics of an SIS Epidemic Model With Immigrants, *Results Control Optim.*, **2**-100007 (2021), 1–13.

[2] R. E. Mickens, Nonstandard Finite Difference Models of Differential Equations, World Scientific, Singapore (1994).

[3] S. Elaydi, An Introduction to Difference Equations, Springer, New York, 3rd ed. (2005).

[4] J. Guckenheimer, P. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Springer-Verlag, New York (1997).

Special Session 3

The impact of prey seasonal breeding on evolutionary predator-prey dynamics

Azmy Ackleh*[†1]

[1] *University of Louisiana Lafayette*
[†] *E-mail: asa5773@louisiana.edu*

Abstract

We extend the evolutionary predator-prey model developed by Ackleh et al. (2019) [1] to incorporate seasonal reproduction in the prey. The extended model distinguishes between breeding and non-breeding seasons, modeling prey reproduction as a period-2 periodic function. We account for the evolution of prey traits driven by toxicant exposure alongside seasonal reproduction dynamics. We establish conditions for population persistence, the existence and stability of periodic solutions, and analyze how seasonal reproduction affects predator-prey interactions under toxicant stress. Our results show that seasonal fluctuations in reproduction influence the evolutionary adaptation of prey to toxicants and, in turn, impact the persistence and stability of both predator and prey populations. Comparisons are made to assess how seasonality modifies the effects of toxicant exposure, identifying parameter regimes where it may either enhance or reduce population densities.

References

[1] A.S. Ackleh, Md I. Hossain, A. Veprauskas and A. Zhang, Persistence and Stability Analysis of Discrete-Time Predator-Prey Models: A Study of Population and Evolutionary Dynamics, Journal of Difference Equations and Applications, 25(2019), 1568-1603.

A two-parameter family of discrete-time predator-prey models: Examining the impact of competition and predation timing

Amy Veprauskas*†1

1 *University of Louisiana Lafayette*
† *E-mail: amy.veprauskas@louisiana.edu*

Abstract

We develop a two-parameter family of discrete-time predator-prey models. The first parameter governs the form of intraspecific competition in the prey population, allowing for either Beverton-Holt or Ricker-type nonlinearities. The second param- eter determines the relative timing between prey density regulation and predation. At the extremes, predation occurs entirely before or entirely after prey density de- pendence. Intermediate values represent asynchronous predation, with a portion of predators feeding at the beginning of the time step and the remainder at the end. We conduct a detailed analysis of this model family to explore how variation in these two aspects of predator-prey interaction influences system dynamics. This framework encompasses the predator-prey models developed in [1, 2] and provides a unifying perspective for assessing how such modeling assumptions affect system behavior.

References

[1] Ackleh, A. S., Hossain, M. I., Veprauskas, A., & Zhang, A. (2019). Persistence and stability analysis of discrete-time predator–prey models: A study of popu- lation and evolutionary dynamics. Journal of Difference Equations and Applica- tions, 25(11), 1568-1603.

[2] Ackleh, A. S., Hossain, M. I., & Veprauskas, A. (2024). Examining the influence of prey dynamics on predator–prey interactions. Journal of Difference Equations and Applications, 30(8), 1194-1221.

Traveling wave dynamics in Wolbachia-infected mosquito populations: Theory, simulation, and release strategy design

Eddy Kwessi*†1

1 *Trinity University*
† *E-mail: ekwessi@trinity.edu*

Abstract

Wolbachia-based vector control strategies rely on the spatial spread of infection through mosquito populations, governed by complex ecological and epidemiological interactions. This work studies the existence, shape, and speed of traveling wave solutions in a lattice-based integro-difference equation (IDE) model of Wolbachia spread. Building on a bistable growth function derived from cytoplasmic incompatibility and maternal transmission dynamics, we characterize conditions for wave propagation across spatial domains using various dispersal kernels (Gaussian, Cauchy, and exponential square root). Theoretical results on wave existence are established via fixed-point arguments, and asymptotic wave speeds are estimated analytically in terms of kernel moments and infection thresholds. Numerical simulations reveal how dispersal mode influences wave speed and shape, and provide practical criteria for minimal release profiles that initiate sustained invasion. We further identify "critical bubbles" — unstable steady states that define threshold conditions for invasion success — and explore the implications of wave stability and environmental robustness. These findings inform the design of spatially optimized, cost-effective release strategies for field deployment of Wolbachia-infected mosquitoes.

Periodical life history strategies and the predator saturation effect

JIM CUSHING*†1

[1] *University of Arizona*
† *E-mail: cushing@arizona.edu*

Abstract

One of first substantial uses of an Allee effect in the modeling of a specific species was done by MG Bulmer in a seminal 1977 paper on periodical insects [1]. The showcase example of a periodical insect are cicadas, some of whose many species exhibit synchronized age cohorts and famous massive adult emergences for reproduction purposes after 13 or 17 years. These species of cicadas are among the most widely studied of all insects; they have long fascinated biologists as the longest-lived insect and because of this cyclic life history strategy. Bulmer utilized discrete time models of Leslie matrix type in his analysis. He incorporated two causal candidates for the cyclic outbreaks and synchronized age cohorts of periodical insects: age-class competition (a kind of intra-specific competitive exclusion principle), which is the typical negative feedback property of density dependence, and the predator saturation effect, which is a positive feedback property of density dependence (i.e. an Allee component). Bulmer concluded that the predator saturation effect cannot alone be the casual mechanism for the life history of periodical insects but can only enhance periodic cycles that are caused by age-class competition. Yet, on the contrary, the widely held consensus among both field and theoretical ecologists is that this life history strategy is indeed caused by the predator saturation effect. In this talk I will discuss how to reconcile this contradiction using Bulmer's modeling methodology.

References

[1] Bulmer, M.G., 1977. Periodical insects. The American Naturalist, 111(982), pp.1099-1117

On the global stability of discrete-time SIS epidemic models: A new analytical framework

OMAIMA SLIMANI*†1, BO ZHENG[2] AND SABER ELAYDI[3]

[1] *Hassan II University, Casablanca, Morocco*
[2] *Guangzhou University, China*
[3] *Trinity University, USA*
† *E-mail: oma96slimani@gmail.com*

Abstract

This paper introduces a novel and unified analytical framework for studying the global dynamics of discrete-time SIS (Susceptible-Infectious-Susceptible) epidemic models incorporating nonlinear recruitment. We focus on the impact of demographic structures, such as Beverton–Holt and Ricker-type recruitment functions, on disease transmission and persistence. A major contribution of this work is the proof of global asymptotic stability (GAS) of the endemic equilibrium when the basic reproduction number $R_0 > 1$—a result that has remained elusive in the general discrete-time non-autonomous setting. Our approach departs from traditional techniques by reducing non-autonomous systems to appropriately defined autonomous limiting systems under minimal assumptions, thus enabling a rigorous analysis of both disease-free and endemic equilibria. Furthermore, we extend our results to periodically forced SIS systems and demonstrate the robustness of our method in such non-autonomous environments. This framework unifies and strengthens previous models while offering new insights into the interplay between recruitment dynamics and epidemic persistence.

References

[1] Saber N. Elaydi , Jim M. Cushing (2025). Discrete Mathematical Models in Population Biology Ecological, Epidemic, and Evolutionary Dynamics. Springer Undergraduate Texts in Mathematics and Technology

[2] E D'Aniello, S Elaydi. The structure of ω-limit sets of asymptotically non-autonomous discrete dynamical systems. Discrete Continuous Dyn Syst, Ser B. 2020;25(3):903–915. doi:10.3934/dcdsb.2019195

[3] K Mokni, S Elaydi, M CH-Chaoui, A. Eladdadi. Discrete evolutionary population models: a new approach. J Biol Dyn. 2020;14(1):454 - 478. doi: 10.1080/17513758.2020.1772997

[4] J. E. Franke and A.-A. Yakubu, *Non-autonomous discrete-time SIS models with application to recurrent epidemics*, Journal of Biological Dynamics, **2**(1), 1–19, 2008.

[5] P. van den Driessche and A.-A. Yakubu, *Discrete SIS models with demographic dynamics*, Journal of Difference Equations and Applications, **24**(8), 1250–1264, 2018.

Special Session 4

Abundance of weird quasiperiodic attractors in piecewise linear discontinuous maps

Davide Radi*†1, Laura Gardini‡2, Iryna Sushko◇3, Noemi Schmitt⋆4 and Frank Westerhoff⁎4

[1] *DiMSEFA, Catholic University of Milan, Italy and VSB TUO, Ostrava, Czech Republic*
[2] *DESP, University of Urbino Carlo Bo, Italy and VSB TUO, Ostrava, Czech Republic*
[3] *DiMSEFA, Catholic University of Milan, Italy and Institute of Mathematics, NAS of Ukraine*
[4] *Dept of Economics, University of Bamberg, Germany*
† *E-mail: davide.radi@unicatt.it and davide.radi@vsb.cz*
‡ *E-mail: laura.gardini@uniurb.it*
◇ *E-mail: sushko@imath.kiev.ua*
⋆ *E-mail: noemi.schmitt@uni-bamberg.de*
⁎ *E-mail: frank.westerhoff@uni-bamberg.de*

Abstract

In this work, we consider a class of n-dimensional, $n \geq 2$, piecewise linear discontinuous maps that can exhibit a new type of attractor, called a weird quasiperiodic attractor (discovered for the firt time in [1] where a financial market model is considered, see also [4]). While the dynamics associated with these attractors may appear chaotic, we prove that a chaotic attractor cannot occur. The considered class of n-dimensional maps allows for any finite number of partitions, separated by various types of discontinuity sets. The key characteristic, beyond discontinuity, is that all functions defining the map have the same real fixed point. These maps cannot have hyperbolic cycles other than the fixed point itself. We consider the two-dimensional case in detail. We prove that in nongeneric cases, the restriction, or the first return, of the map to a segment of straight line issuing from the fixed point is reducible to a piecewise linear circle map (see, e.g., [3]). The generic attractor, different from the fixed point, is a weird quasiperiodic attractor, which may coexist with other attractors or attracting sets. We illustrate the existence of these attractors through numerous examples, using functions with different types of Jacobian matrices, as well as with different types of discontinuity sets. In some cases, we describe possible mechanisms leading to the appearance of these attractors. We also give examples in the three-dimensional space. Several properties of this new type of attractor remain open for further investigation, see also [2].

References

[1] L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, *On the limits of informationally efficient stock markets: New insights from a chartist-fundamentalist model.* Forthcomnig in International Review of Financial Analysis. (2025) https://arxiv.org/abs/2410.21198.

[2] L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, *On the emergence and properties of weird quasiperiodic attractors,* Submitted for publication. (2025) https://arxiv.org/pdf/2503.11264.

[3] L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, *Dynamics of 1D discontinuous maps with multiple partitions and linear functions having the same fixed point. An application to financial market modeling*, Working Paper, University of Bamberg (2025) (DOI: 10.48550/arXiv.2503.20449, available at https://arxiv.org/abs/2503.20449).

[4] L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, *How risk aversion may shape the dynamics of stock markets: a chartist-fundamentalist approach,* Working Paper, University of Bamberg (2025).

Weird quasiperiodic attractors in a family of Belykh maps

Laura Gardini*†1, Davide Radi‡2 Noemi Schmitt◇3, Frank Westerhoff⋆3 and Iryna Sushko⁎4

[1] *DESP, University of Urbino Carlo Bo, Italy Dept of Finance, VSB - Technical University of Ostrava, Ostrava, Czech Republic*
[2] *DiMSEFA, Catholic University of Milan, Italy Dept of Finance, VSB - Technical University of Ostrava, Ostrava, Czech Republic*
[3] *Dept of Economics, University of Bamberg, Germany*
[4] *DiMSEFA, Catholic University of Milan, Italy*
† *E-mail: laura.gardini@uniurb.it*
‡ *E-mail: davide.radi@unicatt.it*
◇ *E-mail: noemi.schmitt@uni-bamberg.de*
⋆ *E-mail: frank.westerhoff@uni-bamberg.de*
⁎ *E-mail: sushko@imath.kiev.ua*

Abstract

We consider the map

$$\begin{cases} x' = x + (g(x) + y) \\ y' = \lambda(g(x) + y) \end{cases}$$

known as Belykh map, where $g(x)$ can be smooth or PW smooth. With suitable choices of $g(x)$ it is topologically conjugate to the Henon map and to a Lozi map. Its peculiarity is that **the Jacobian determinant is λ, in any point of the phase plane**, and it is useful to study hyperbolic attractors [1,2,3].

We focus on the PWL case, with $g(x) = \beta x$ in the L partition and $g(x) = -\beta x$ in the R partition, with $-1 < \lambda < 1$ and as discontinuity set some different options. Thus, the map is given by

$$B_1 = \begin{cases} B_L : \begin{cases} x' = (1+\beta)x + y \\ y' = \lambda(\beta x + y) \end{cases} & J_L = \begin{bmatrix} 1+\beta & 1 \\ \lambda\beta & \lambda \end{bmatrix} \\ \\ B_R : \begin{cases} x' = (1-\beta)x + y \\ y' = \lambda(-\beta x + y) \end{cases} & J_R = \begin{bmatrix} 1-\beta & 1 \\ -\lambda\beta & \lambda \end{bmatrix} \end{cases}$$

and it belongs to a class of two-dimensional discontinuous maps recently defined, that leads to a new kind of attractor, called weird quasiperiodic attractor (WQA) [4], [5], [6]. Their existence does not depend on the kind of discontinuity sets considered, but the shape and properties of WQAs depend on the discontinuity set. We will consider the straight line $x = 1$, two straight lines $x = \pm 1$ and the circle $x^2 + y^2 = 1$.

References

[1] V. Afraimovich and S. Hsu, Lectures on chaotic dynamical systems, American Mathematical Societyand International Press, 2003.

[2] V. Belykh, Chaotic and strange attractors of a two-dimensional map, Mat. Sb. 186(3) (1995), pp.311–326.

[3] V. N. Belykh, N. V. Barabash & D. A. Grechko (2023): Existence proofs for strange attractors in piecewise-smooth nonlinear Lozi-H´enon and Belykh maps, Journal of Difference Equations and Applications, DOI: 10.1080/10236198.2023.2193653.

[4] L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, On the limits of informationally efficient stock markets: New insights from a chartistfundamentalist model. International Review of Financial Analysis, to appear (2025).

[5] L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, On the emergence and properties of weird quasiperiodic attractors, Submitted for publication. (2025) https://arxiv.org/pdf/2503.11264.

[6] L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, How risk aversion may shape the dynamics of stock markets: a chartist-fundamentalist approach. (2025) https://arxiv.org/pdf/2504.04778.

Critical structures and attractor evolution in a noninvertible piecewise smooth map under quasiperiodic forcing

Jicheng Duan*†1, Laura Gardini[2], Zhouchao Wei[1], Iryna Sushko[3] and Yiwen Chen[4]

[1]*School of Mathematics and Physics, China University of Geosciences, China*
[2]*Department of Economics, Society and Politics, University of Urbino Carlo Bo, Italy*
[3]*Institute of Mathematics, National Academy of Sciences of Ukraine, Ukraine*
[4]*School of Mathematics and Physics, Lanzhou Jiaotong University, China*
†*E-mail: jichengduan9708@163.com*

Abstract

This paper investigates the formation and evolution of strange nonchaotic attractors (SNAs) in a noninvertible piecewise smooth map subjected to quasiperiodic forcing. The model is constructed as a two-dimensional skew-product system, combining a one-dimensional piecewise smooth base map with an irrational rotation. The presence of both a smooth critical curve and a discontinuity line generates multiple critical structures that shape the forward dynamics. By exploring representative parameter regimes, four distinct scenarios are identified: (i) interactions between smooth critical curves and absorbing regions; (ii) boundary changes due to the discontinuity; (iii) the combined influence of smooth and discontinuous critical sets; and (iv) coexistence and bifurcation cascades of high-period tori. For each scenario, numerical and analytical results demonstrate how the interplay between critical curves and preimage regions governs transitions from smooth tori to SNAs, leading to foldings, band splittings, and the formation of holes in the basin. Overall, the study shows that the combined effect of smooth folds and discontinuities determines the structure, evolution, and boundaries of attractors in this class of quasiperiodically forced noninvertible maps, extending previous results for purely smooth cases.

Robust chaos in piecewise-linear maps

Indranil Ghosh*†[1] and David Simpson[1]

[1] *Massey University, New Zealand*
† *E-mail: i.ghosh@massey.ac.nz*

Abstract

The two-dimensional border-collision normal form is a family of piecewise-linear maps with complex bifurcation structures. In this talk, I will show how renormalisation can be utilised to better understand these structures. I will focus on the occurrence of robust chaos and how the topology (number of connected components) of the chaotic attractor changes in various regions where the map is orientation-preserving, orientation-reversing, or noninvertible. This is done via numerical evaluation of the renormalisation operator. Broadly speaking, renormalisation involves showing that, for some members of a family of maps, a higher iterate or induced map is conjugate to a different member of this family. I will show how the results obtained through renormalisation compare to those obtained with bruteforce numerical computations using an algorithm of Eckstein and Avrutin. This method is based on computing the greatest common divisors in return times and agrees very well with our renormalisation approach. I will also talk about how to verify Devaney chaos in the orientation-preserving setting, and how robust chaos can be demonstrated in maps with more than two dimensions in terms of a positive Lyapunov exponent.

Weird quasiperiodic attractors: Properties and mechanisms of appearance

Iryna Sushko*†1, Laura Gardini‡2, Davide Radi◇3, Noemi Schmitt⋆4 and Frank Westerhoff⁎4

[1] *Institute of Mathematics, NAS of Ukraine*
[2] *DESP, University of Urbino Carlo Bo, Italy*
[3] *DiMSEFA, Catholic University of Milan, Italy*
[4] *Dept of Economics, University of Bamberg, Germany*
† *E-mail: sushko@imath.kiev.ua*
‡ *E-mail: laura.gardini@uniurb.it*
◇ *E-mail: davide.radi@unicatt.it*
⋆ *E-mail: noemi.schmitt@uni-bamberg.de*
⁎ *E-mail: frank.westerhoff@uni-bamberg.de*

Abstract

Considering a financial market model defined by a 2D piecewise linear map [1], we discovered a new type of attractor, which we call a weird quasiperiodic attractor (WQA). It has a rather complex geometric structure and other interesting properties that are worth understanding better. To this end, in [2] we consider a simpler map that can also possess WQAs, namely, a 2D discontinuous piecewise linear map F with a single discontinuity line dividing the phase plane into two partitions, where two different homogeneous linear maps are defined. Map F depends on four parameters – the traces and determinants of the two Jacobian matrices. In the parameter space of map F, we obtain specific regions associated with the existence of weird quasiperiodic attractors, describe some characteristic properties of these attractors, and explain one possible mechanism for their emergence.

References

[1] L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, On the limits of informationally efficient stock markets: New insights from a chartist-fundamentalist model, arxiv.org/abs/2410.21198.

[2] L. Gardini, D. Radi, N. Schmitt, I. Sushko and F. Westerhoff, On the emergence and properties of weird quasiperiodic attractors, arxiv.org/abs/2503.11264.

Noninvertibility and bifurcation phenomena in a four-partitions piecewise linear map

Wirot Tikjha*†1

1 *Faculty of Sciences and Technology, Pibilsongkram Rajabhat University, Thailand*
† *E-mail: wirottik@psru.ac.th*

Abstract

This presentation investigates the dynamics of a four-quadrant piecewise linear (PWL) map defined by distinct linear rules in each quadrant. We extend the framework of bifurcations and border-collision bifurcations (BCBs) to systems with multiple critical lines ($x = 0$ and $y = 0$), emphasizing the role of bifurcation transitions. Novel phenomena such as multistability and noninvertibility are analyzed.

References

[1] Tikjha, W., & Gardini, L. (2020). Bifurcation sequences and multistability in a two-dimensional piecewise linear map. International Journal of Bifurcation and Chaos, 30(6), 2030014. https://doi.org/10.1142/S0218127420300141

[2] Gardini, L., & Tikjha, W. (2019). Role of the virtual fixed point in the center bifurcations in a family of piecewise linear maps. International Journal of Bifurcation and Chaos, 29(12), 1930041. https://doi.org/10.1142/S0218127419300410

[3] Gardini, L., & Tikjha, W. (2020). Dynamics in the transition case invertible/non-invertible in a 2D Piecewise Linear Map. *Chaos, Solitons & Fractals*, *137*, 109813.

Heterogeneous environmental attitudes and economic dynamics in an OLG framework

Fausto Cavalli[1], Ahmad Naimzada[†1] and Nicolò Pecora[*‡2]

[1] *DEMS, University of Milano, Bicocca, Italy*
[2] *DISES, Catholic University of Piacenza, Italy*
[†] *E-mail: ahmad.naimzada@unimib.it*
[‡] *E-mail: nicolo.pecora@unicatt.it*

Abstract

This paper develops an overlapping generations (OLG) model in which the evolution of pollution and the formation of environmental concern are endogenously determined. Individuals, heterogeneous in their environmental attitudes, contribute to pollution while also making economic choices that influence the transmission of environmental concern across generations. To capture this phenomenon, we introduce a switching mechanism that allows agents to modify their environmental concern on the basis of a sentiment index which captures the mental framework, and the complex heuristics involved in agents' reasoning. The model we end up with is able to generate multiple steady states characterized by persistent high or low levels of pollution, thus reflecting the general attitude of agents in the economy. Further, the switching mechanism is also augmented to account for feedback effects from the environment, namely by considering the pollution dynamics. We study the resulting dynamics and show that complex endogenous dynamics may also emerge, highlighting the role of the interaction among heterogeneous agents, environmental and macroeconomic conditions

References

[1] Bezin, E. (2015). *A cultural model of private provision and the environment.* Journal of Environmental Economics and Management, 71, 109-124.

[2] John, A., & Pecchenino, R. (1994). *An Overlapping Generations Model of Growth and the Environment.* Economic Journal, 104(427), 1393-1410.

[3] Schumacher, I., & Zou, B. (2008). *Pollution perception: A challenge for intergenerational equity.* Journal of Environmental Economics and Management, 55(3), 296-309.

Two-player Yorke's game of survival in chaotic transients

Miguel A.F. Sanjuán*†1

[1] *Nonlinear Dynamics, Chaos and Complex Systems Group, Departamento de Física, Universidad Rey Juan Carlos, Tulipán s/n, 28933 Móstoles, Madrid, Spain*
† *E-mail: miguel.sanjuan@urjc.es*

Abstract

We present a novel two-player game in a chaotic dynamical system where players have opposing objectives regarding the system's behavior. The game is analyzed using a methodology from the field of chaos control known as partial control. Our aim is to introduce the utility of this methodology in the scope of game theory. These algorithms enable players to devise winning strategies even when they lack complete information about their opponent's actions. To illustrate the approach, we apply it to a chaotic system, the logistic map. In this scenario, one player aims to maintain the system's trajectory within a transient chaotic region, while the opposing player seeks to expel the trajectory from this region. The methodology identifies the set of initial conditions that guarantee victory for each player, referred to as the *winning sets*, along with the corresponding strategies required to achieve their respective objectives. This is joint work with Gaspar Alfaro and Rubén Capeáns from URJC, Madrid, Spain.

References

[1] J. Aguirre, F. d'Ovidio, and M. A. F. Sanjuán, *Controlling chaotic transients: Yorke's game of survival*, Phys. Rev. E **69** (2004), 016203.

[2] Juan Sabuco, Miguel A. F. Sanjuán and James A. Yorke, *Dynamics of Partial Control*, Chaos **22** (2012), 047507-1–047507-9.

[3] Rubén Capeáns, Juan Sabuco, and Miguel A. F. Sanjuán, *A new approach of the partial control method in chaotic systems*, Nonlinear Dynamics **98** (2019), 873–887.

[4] Rubén Capeáns and Miguel A. F. Sanjuán, *Beyond partial control: Controlling chaotic transients with the safety function*, Nonlinear Dynamics **107** (2022), 2903–2910.

[5] Gaspar Alfaro, Rubén Capeáns and Miguel A. F. Sanjuán, *Two-Player Yorke's Game of Survival in Chaotic Transients*, (2025), https://arxiv.org/abs/2501.12188

Uncertainty, environmental control and free entry equilibria in evolutionary oligopolies

Fausto Cavalli*†1, Ahmad Naimzada‡1 and Marina Pireddu◇2

[1] Dept. of Economics, Management and Statistics, University of Milano-Bicocca, Milano, Italy
[2] Dept. of Mathematics and its Applications, University of Milano-Bicocca, Milano, Italy
† E-mail: fausto.cavalli@unimib.it
‡ E-mail: ahmad.naimzada@unimib.it
◇ E-mail: marina.pireddu@unimib.it

Abstract

We revisit the classic problem of firms' distribution within an economy (see e.g., [3]) by expanding the traditional framework that includes two markets, namely an oligopoly and "the rest of the world", while considering the effects of environmental taxation and firms' uncertainty about it. In the rest of the world, firms achieve an exogenously determined profit level, $\bar{\pi}$. In the oligopoly market, firms engage in a Cournot competition ([1, 4]). Each firm $i = 1, \dots, n$ produces a quantity q_i of a homogeneous good, emitting an amount eq_i of pollutants during the manufacturing process. The normalized parameter $e \in [0, 1]$ describes the strength of abatement efforts by the firms. The profits the firms achieve depend on a possible penalty or reward that a regulator chooses, based on the environmental situation. The regulator intervenes with the aim of keeping emissions under control and promoting an improvement in environmental conditions. Following [2], we assume that the regulator cannot measure the precise amount of pollution in the environment, and it is also impossible to estimate the pollution emissions of each firm. The decision to impose a tax when the environmental situation is poor or to introduce incentives when the pollution level is low depends on the comparison between the measured pollution level and an environmental standard E, adjusted by a penalty/reward rate $\theta > 0$. Firms do not know the measurement the regulator will obtain and, therefore, whether they will be charged a penalty or receive a subsidy. The resulting profits for each firm $i = 1, \dots, n$ can be described by the stochastic expression

$$\pi_i = q_i\, p\left(\sum_{j=1}^{N} q_j\right) - c(q_i) - \theta\left(\varepsilon_i \sum_{j=1}^{N} e\, q_j - E\right),$$

where ε is a random variable with unitary mean and standard deviation δ, while $\sum_{i=1}^{n} eq_i$ are the aggregate emissions. Firms face the goal of maximizing expected profits while minimizing the uncertainty about them. This leads to a multi-objective optimization problem that can be scalarized by introducing the relative relevance α, which is assigned to minimizing the standard deviation in the optimization process. Each firm, for $i = 1, \dots, n$, then plays a game in which it chooses the quantity to produce based on the maximization of the weighted average between the expected profits and the (negative) standard deviation

$$\varphi_i = \mathbb{E}(\pi_i) - \alpha D(\pi_i) = q_i p\left(\sum_{j=1}^{N} q_j\right) - c(q_i) - \theta\left(\sum_{j=1}^{N} e\, q_j - E\right) - \alpha\,\theta\left(\sum_{j=1}^{N} e\, q_j\right)\delta_i.$$

Firms choose to operate in the oligopolistic market or in the rest of the world depending on the expected profitability that can be achieved in each market. Due to the uncertainty characterizing profits, when a firm must select the market in which to operate, the distribution of firms between the two markets is not accurately known and is described by a random variable, about which firms only know the expected value, ω, i.e., the probability that a firm will choose market 1, i.e. the oligopolistic one. The expected share of firms choosing to operate in market 1 is described by an evolutionary selection mechanism based on discrete replicator dynamics, regulated by fitness measures corresponding to the expected profits in each market, namely $\pi^*(\omega)$ and $\bar{\pi}$, respectively. Accordingly, for $t \in \mathbb{N}$, if $\omega_t > 0$, the evolution of the probability of choosing market 1 is ruled by:

$$\omega_{t+1} = \frac{\omega_t \exp(\beta\pi^*(\omega_t))}{\omega_t \exp(\beta\pi^*(\omega_t)) + (1-\omega_t)\exp(\beta\bar{\pi})},$$

which is the model under investigation. We study it both statically and dynamically, showing the evolution of possible distributions among markets and investigating the stability of the dynamics, focusing on the role of the weight α assigned to the uncertainty.

References

[1] G.I. Bischi, C. Chiarella, M. Kopel, F. Szidarovszky, Nonlinear Oligopolies. Stability and Bifurcations, Springer-Verlag, Berlin, 2010.

[2] A. Matsumoto, F. Szidarovszky, N-firm oligopolies with pollution control and random profits, Asia-Pac. J. of Reg. Sci., 6, 1017–1039, 2022.

[3] M. Polo, Entry games and free entry equilibria, IEFE Working Paper - The Center for Research on Energy and Environmental Economics and Policy at Bocconi University, 87 (2016).

[4] N. Singh, X. Vives, Price and quantity competition in a differentiated duopoly, Rand J. Econ. 15, 546554, 1984.

On systems with many periodic solutions

Inese Bula*†[1,2]

[1] *Department of Mathematics, University of Latvia*
[2] *Institute of Mathematics and Computer Science, University of Latvia*
† *E-mail: inese.bula@lu.lv*

Abstract

We consider the systems of first order piecewise linear difference equations:

$$\begin{cases} x_{n+1} = |x_n| - y_n - b, \\ y_{n+1} = x_n - |y_n| - d, \end{cases} \quad (1) \qquad \begin{cases} x_{n+1} = |x_n| - y_n - b, \\ y_{n+1} = x_n - |y_n| + d, \end{cases} \quad (2)$$

$n = 0, 1, 2, ..., (x_0, y_0) \in \mathbf{R}^2$, where parameters b and d are any positive real numbers.

The system (1) there exist an unstable equilibrium $(d; -b)$. It has been shown that there are no solutions with period 2, 3 and 4, but depending on the values of parameters b and d there are solutions with periods 5, 6, 7, 11, 12, 13, 16, 17, 18, 19, 20, 24, 27, 30, 36. The obtained results are published in the article [1]. The system (2), if $2d - b \leq 0$, then there exist an unstable equilibrium $(\frac{-2b-d}{5}; \frac{2d-b}{5})$. It has been shown that there are no solutions with period 2, but depending on the values of parameters b and d there are solutions with periods 3, 4, 7, 10, 11, 14, 17, 20, 24. In the general case, both of the above systems are special cases of the system (in, E.A. Grove and G. Ladas, 2005):

$$\begin{cases} x_{n+1} = |x_n| + a\, y_n + b, \\ y_{n+1} = \ x_n \ + c\, |y_n| + d, \end{cases} \quad n = 0, 1, 2, ..., (x_0, y_0) \in \mathbf{R}^2. \qquad (3)$$

It is clear that not all cycles have been found of systems (1) and (2), and it is also not clear what cycles exist and whether there is any limit to the size of a cycle. Perhaps the article [3] can provide answers to these questions about system (2), but determining higher-order periodic cycles could be a serious problem. Are there other special cases of system (3) with many large cycles?

References

[1] Bula, I., Sīle, A., *About a System of Piecewise Linear Difference Equations with Many Periodic Solutions*, S. Olaru et al. (eds.), Difference Equations, Discrete Dynamical Systems and Applications, ICDEA2022, Springer Proceedings in Mathematics & Statistics **444** (2024), 29 - 50.

[2] Cima, A., Gasull, A., Manosa, V., Manosas, F., *Invariant Graphs and Dynamics of a Family of Continuous Piecewise Linear Planar Maps*, Qualitative Theory of Dynamical Systems, **24**, 70, 103 pages, 2025.

Special Session 5

Discrete μ-dichotomy spectrum

Álvaro Castañeda*†1, Néstor Jara[1] and Claudio A. Gallegos[1]

[1] *Universidad de Chile, Chile*
† *E-mail: castaneda@uchile.cl*

Abstract

We develop spectral theorems for nonautonomous linear difference systems under various μ-dichotomy conditions, emphasizing the nonuniform setting. This leads to surprising behaviors that motivate the introduction of two new properties for nonautonomous systems and a refined notion of nonuniform μ-dichotomy spectrum, situated between classical and slow spectra. We also propose a conjecture that supports spectral results in this new context. Finally, using optimal ratio maps, we demonstrate that the nonuniform exponential dichotomy spectrum is not invariant under weak kinematic similarity, challenging a belief held in recent literature.

Keywords: Nonautonomous difference equations, Nonautonomus hyperbolicity, Nonuniform dichotomy, Nonuniform dichotomy spectrum, Kinematic similarity

Admissibility and robustness of exponential trichotomy for discrete nonautonomous dynamics

Davor Dragičević[1], Adina Luminita Sasu*†[2,3] and Bogdan Sasu[2,3]

[1] *University of Rijeka, Croatia*
[2] *West University of Timisoara, Romania*
[3] *Academy of Romanian Scientists, Romania*
† *E-mail: adina.sasu@e-uvt.ro*

Abstract

We present conditions for the robustness of the uniform exponential trichotomy in the sense of Palmer, Sacker and Sell (see [2-5]). Starting from admissibility criteria obtained in [4], we present a constructive method of exploring the persistence of the uniform exponential trichotomy of discrete nonautonomous systems in the presence of perturbations. We discuss the optimality of the perturbations that can be employed - which in this case belong to $\ell^1(\mathbb{Z}, \mathcal{B}(X))$, as well as the fact that this trichotomy notion is not robust under other categories of perturbations. Finally, inspired by the method recently introduced in [1] we point out open problems and new directions.

Keywords: Admissibility, Exponential trichotomy, Robustness, Nonautomomous system

References

[1] D. Dragičević, A. L. Sasu, B. Sasu, Robustness of uniform exponential trichotomy: an input-output approach, J. Differential Equations 439 (2025), 113409.

[2] K. J. Palmer, Shadowing and Silnikov chaos, Nonlinear Anal. 27 (1996), 1075-1093.

[3] R. J. Sacker, G. R. Sell, Existence of dichotomies and invariant splittings for linear differential systems III, J. Differential Equations 22 (1976), 497-522.

[4] A. L. Sasu, B. Sasu, Discrete admissibility and exponential trichotomy of dynamical systems, Discrete Contin. Dyn. Syst. 34 (2014), 2929-2962.

[5] A. L. Sasu, B. Sasu, Exponential trichotomy and (r, p)-admissibility for discrete dynamical systems, Discrete Contin. Dyn. Syst. Ser. B 22 (2017), 3199-3220.

Differentiable linearization for hyperbolic and partially hyperbolic diffeomorphisms without any non-resonant condition

Wenmeng Zhang*†1

[1] *Chongqing Normal University, China*
† *E-mail: wmzhang@cqnu.edu.cn*

Abstract

We will first study the linearization problem for hyperbolic diffeomorphisms in R^n and in Banach spaces, respectively, and show that the conjugacy is differentiable at the fixed point without any non-resonant condition. Then, we further study this problem for partially hyperbolic diffeomorphisms in R^n, i.e., with center direction, and show that, without any non-resonant condition, the conjugacy is differentiable at the center manifold with a continuous derivative.

Exponential dichotomy and generalized Pugh's global linearization theorem

Yonghui Xia*†1

[1]*Foshan University, China*
†*E-mail: xiadoc@163.com*

Abstract

This talk is to present some recent advance in exponential dichotomy and its application to the generalized Pugh's global linearization theorem.

Generalized Green function, Lipschitz shadowing and generalized limit shadowing for infinite-dimensional systems

Linfeng Zhou*†1

[1]Sichuan University, China
†E-mail: zlf63math@163.com

Abstract

In this talk, we will introduce our work on an extension of the generalized Green function for invertible systems to non-invertible cases, the Lipschitz shadowing under weaker conditions and a generalized limit shadowing property for infinite-dimensional systems.

1:4 resonance, arnold tongues and Marotto's chaos of a discrete reduced Lorenz system

Jiyu Zhong*†[1]

[1] *Hainan University, NO.58 renmin road haikou city hainan province, 570228, China*
† *E-mail: matzhjy@sina.com*

Abstract

In this paper, firstly, asymptotically embedding the 4th iterate of the normal form of a discrete reduced Lorenz system into a flow, we present rigorously the bifurcation sequence as the parameter varies around the 1:4 strong resonance point, from which an Arnold tongue corresponding to a rotation number 1/4 is obtained. Then, by the theory of normal form, we give theoretically the Arnold tongues in the weak resonances such that the system possesses two periodic orbits on the stable invariant closed curve generated from the Neimark-Sacker bifurcation. Furthermore, we obtain rigorously the parameter conditions under which the system has the chaotic behavior in Marotto's sense. Finally, by numerical simulations, not only our results are verified, but also some new and interesting dynamical behaviors are discovered.

Keywords: 1:4 resonance, Arnold tongue, Periodic orbit, Normal form, Chaotic attractor

Takens theorem for nonautonomous partially hyperbolic dynamical systems

Xiao Tang*†[1], Wenmeng Zhang[1] and Davor Dragičević[2]

[1] *Chongqing Normal University, China*
[2] *University of Rijeka, Croatia*
† *E-mail: mathtx@163.com*

Abstract

Takens theorem for a partially hyperbolic dynamical system provides a normal linearization along the center manifold. In this paper, we give the nonautonomous version of Takens theorem under non-resonance conditions formulated in terms of the dichotomy spectrum. In our proof, one difficulty is solving homological equations for the normal form theory which involve a center variable, while another difficulty is finding the dichotomy spectrum of a certain matrix cocycle that is block lower triangular. In order to overcome those difficulties, in comparison with the autonomous case, we need an additional term in the (nonautonomous) non-resonance conditions to guarantee certain spectral gap conditions. This additional term disappears naturally in the autonomous case.

Concave operators: extensions and stability of associated contractions with applications to discrete dynamics

Laurian Suciu*†1, Catalin Badea[2] and Vladimir Müller[3]

[1] *"Lucian Blaga" University of Sibiu, Sibiu, Romania*
[1] *Univ. Lille, CNRS, UMR 8524, Laboratoire Paul Painlevé, France*
[2] *Czech Academy of Sciences Praha, Czech Republic*
† *E-mail: laurians2002@yahoo.com*

Abstract

This talk investigates the class of concave operators T on Hilbert spaces through the lens of discrete dynamical behavior and operator extension theory. Building on analogies with Brownian-type unitary extensions of 2-isometries due to Agler and Stankus, we construct special concave extensions for such operators, encoded in an upper-triangular operator matrix.

While the extension retains concavity, the associated contraction C becomes an important object of study, particularly with regard to its stability properties. This perspective allows us to link robust properties of T to dynamical features of C, including contractive recurrence and convergence phenomena.

Several classes of examples illustrate how this framework captures meaningful aspects of discrete dynamics and provides tools for stability analysis.

Stability of discrete dynamics via trajectory methods with Banach sequence spaces

DAVOR DRAGIČEVIĆ[1], ADINA LUMINITA SASU[2], BOGDAN SASU[2] AND ANA SIRIANTU[*†2]

[1] *University of Rijeka, Croatia*
[2] *West University of Timisoara, Romania*
[†] *E-mail: ana.siriantu@e-uvt.ro*

Abstract

We describe the nonuniform exponential stability of a discrete variational system in terms of the belonging of its trajectories to Banach sequences spaces from some well-chosen classes. As consequences we deduce characterizations for uniform exponential stability. Our approach has as a starting point a method introduced in (1) based on ergodic theory arguments and relies on the theory of Banach sequence spaces. As applications, we explore the roughness of nonuniform exponential stability in the presence of perturbations and discuss consequences in the uniform case. Furthermore, we present characterizations for exponential stability of skew-product flows.

Keywords: Exponential stability, Discrete variational system, Trajectory, Banach sequence space

Special Session 6

Periodic dynamics of a switching discrete system of reaction-diffusion equations

RUIBIN JIANG[1], YUNFENG LIU[1], YUMING CHEN[2] AND ZHIMING GUO*†1

[1] *Guangzhou University, China*
[2] *Wilfrid Laurier University, Canada*
† *E-mail: guozm@gzhu.edu.cn*

Abstract

Periodicity is a common phenomenon in biological systems. In this talk, we firstly characterize it with an abstract switching discrete system of reaction-diffusion equations. Under certain conditions, we obtain the critical thresholds for the time interval and period, denoted by N^* and T^*, respectively. Specifically, when the time interval $N > N^*$ (or, the period $T > T^*$), the system admits a unique globally asymptotically stable T-periodic solution. Conversely, if $N \geq N^*$ (or $T \geq T^*$), the trivial equilibrium is globally asymptotically stable. We further demonstrate the applicability of our theoretical framework with two biologically motivated examples: a mosquito population suppression model based on the Sterile Insect Technique and a harvesting model in fishery management. Numerical simulations are also conducted to validate our analytical findings.

Traveling waves for bistable monotone maps

Jian Fang*†1

[1] *Harbin Institute of Technology (HIT), China*
† *E-mail: jfang@hit.edu.cn*

Abstract

In this talk, I will discuss the existence and nonexistence of traveling waves for a class of monotone maps with the bistability structure.

Keywords: Traveling waves, Bistable maps

Classification of a class of singular matrix difference equations of mixed order

Huaqing Sun*†1 and Li Zhi[2]

[1] *Jiangsu Normal University, China*
[2] *Harbin Institute of Technology, China*
† *E-mail: sunhuaqing_2@163.com*

Abstract

This talk is concerned with singular matrix difference equations of mixed order. The existence and uniqueness of initial value problems for these equations are derived, and then the classification of them is obtained. An equivalent characterization of this classification is given in terms of the number of linearly independent square summable solutions of the equation. The influence of off-diagonal coefficients on the classification is illustrated. In particular, two limit point criteria are established in terms of coefficients of the equation.

Eigenvalues of discrete second order difference operator and its nonlinear application

Chenghua Gao*†[1] and Ruyun Ma[2]

[1] *Northwest Normal University, China*
[2] *Xidian University, China*
† *E-mail: gaokuguo@163.com*

Abstract

In this talk, we present some recent results on the spectra of linear difference operators. In particular, we investigate the spectra of second-order difference operators with sign-changing weights, linear spectral parameters, and nonlinear parameters appearing in the boundary conditions. Furthermore, as an application, we also study the existence of solutions to a class of nonlinear problems.

Positive solutions of partial discrete Kirchhoff type problems

Feng Xiong*†[1,2], Wentao Huang[2] and Yonghui Xia[1]

[1]*School of Mathematical Sciences, Zhejiang Normal University, China*
[2]*College of Mathematics and Satistics, Guangxi Normal University, China*
†*E-mail: xiongfengq1@163.com*

Abstract

In this paper, by using critical point theory, we study the infinitely many solutions of partial discrete Kirchhoff type problems. Moreover, we acquire some sufficient conditions for the existence of positive solutions to the boundary value problems by using the strong maximum principle. As far as we know, this is the research of partial discrete Kirchhoff type problems for the first time. Finally, our major results are explained with four examples.

Special Session 7

Electronic encyclopedia for powers of generating functions

Dmitry Kruchinin*†[1] and Arsen Merinov[1]

[1] *Tomsk State University of Control Systems and Radioelectronics, Russia*
† *E-mail: kruchinindm@gmail.com*

Abstract

Generating functions have application in various mathematical disciplines such as combinatorics, statistics, number theory, polynomial theory, and etc. In the paper we consider an automated search system for generating functions of two variables. The system is consist of a modul of the Maxima programm and online service.

An important problem associated with generating functions is the problem of finding the coefficients of the powers of generating functions. We show how it is possible to use the system for obtaining explicit expressions for the composition of generating functions of two variables, coefficients of reciprocal and compositional inverse generating functions of two variables.

This research was funded by the Russian Science Foundation, grant number 22-71-10052.

Compositions of AND/OR tree structures based on generating functions

Yuriy Shablya[†1], Vladimir Kruchinin[1] and Dmitry Kruchinin[*‡1]

[1] *Tomsk State University of Control Systems and Radioelectronics, Russia*
[†] *E-mail: shablya-yv@mail.ru*
[‡] *E-mail: kruchinindm@gmail.com*

Abstract

Generating functions play a key role in combinatorics when solving problems of enumerating finite sets. At the same time, the mathematical apparatus of AND/OR trees allows us to develop algorithms for generating elements of such combinatorial sets. If we consider two combinatorial sets A_n and B_n, which contain combinatorial objects of size n, then based on the sequences of their cardinality $|A_n|$ and $|B_n|$ the following two generating functions are formed:

$$\sum_{n\geq 0} |A_n|\, x^n = A(x), \quad \sum_{n\geq 0} |B_n|\, x^n = B(x).$$

Performing operations on generating functions $A(x)$ and $B(x)$ forms a new combinatorial set associated with the elements of the combinatorial sets A_n and B_n. This report discusses the following operations on generating functions: addition $A(x)+B(x)$, subtraction $A(x)-B(x)$, multiplication $A(x)\cdot B(x)$, the k-th power $A(x)^k$. For each case, the formation of combinatorial sets is considered, as well as the derivation of recurrent formulas for their enumeration and the construction of corresponding compositions of AND/OR tree structures. In addition, the report examines the solution of enumeration problems and construction of combinatorial generation algorithms for combinatorial sets that are defined by a generating function $A(x)$ satisfying the equation

$$A(x) = \sum_{i=0}^{m} P_i(x)\cdot A(x)^i, \quad P_i(x) = \sum_{j=0}^{n_i} p(i,j)\, x^j.$$

Nonlinear elliptic Dirichlet boundary value problem on generalized time scale

Shalmali Bandyopadhyay*†1

[1] *The University of Tennessee at Martin, United States*
† *E-mail: sbandyo5@utm.edu*

Abstract

This paper investigates the existence and uniqueness of solutions to nonlinear elliptic Dirichlet boundary value problems on n-dimensional generalized time scales. We extend previous work that focused primarily on two-dimensional settings to a more general framework, addressing the gap in literature stemming from an incomplete understanding of multi-variable time scale calculus. Our focus is on the boundary value problem $-Tu + f(x, u(x)) = 0$ in Ω with $u = 0$ on $\partial\Omega$, where Ω is a product space formed by n generalized time scales. Under the assumption that f satisfies a Lipschitz condition we establish the existence and uniqueness of solutions using contraction mapping theorem. Our theoretical framework unifies and extends classical elliptic PDE theory, offering a powerful approach for studying steady-state phenomena in hybrid or irregular temporal structures that arise in various physical and biological systems.

Ulam type stability for the Beverton-Holt model

MASAKAZU ONITSUKA*†1

[1] *Okayama University of Science (OUS), 1-1 Ridai-cho, Okayama 700-0005, Japan*
† *E-mail: onitsuka@ous.ac.jp*

Abstract

This study is based on joint work with Professor D.R. Anderson from Concordia College, USA. Ourstudy explores the conditional Ulam stability (or conditional Hyers-Ulam stability) of a first-order nonlinear h-difference equation, specifically the Beverton-Holt model. A key challenge in nonlinear Ulam stability analysis is determining the limits for both the size of the perturbation and the initial population. Using a novel approach, we've derived explicit expressions for the optimal lower bound of the initial value region and the upper bound of the perturbation amplitude. This improves upon the precision of prior research. Furthermore, we've obtained a sharper Ulam stability constant, which measures the error between true and approximate solutions, showing improved stability. We prove that this constant depends on the step-size h and the growth rate, but not on the carrying capacity. We also provide detailed examples that illustrate the applicability and sharpness of our conditional stability results.

Keywords: Nonlinear difference equation, Conditional Ulam stability, Ulam stability constant

Discrete boundary value problem on the integer lattice for a two-dimensional linear recurrence

Alexander Lyapin*†1

[1]*Siberian Federal University (SibFU), 79 Svobodny pr., 660041 Krasnoyarsk, Russia*
†*E-mail: alexplyapin@gmail.com*

Abstract

Westudy a boundary value problem for a two-dimensional linear difference equation with constant coefficients. The problem is formulated in a rectangular subset of the integer lattice and involves specifying the solution on a set of “boundary” points structured in vertical and horizontal directions. We analyze the solvability conditions of the discrete equation by constructing the associated finite linear system and examining the determinant of its coefficient matrix, which has a block tridiagonal form. A necessary and sufficient condition for the existence and uniqueness of the solution is established in terms of the non-vanishing of this determinant.

In addition, we present a symbolic approach to compute generating functions of solutions using characteristic polynomials and sliced initial data series. This generalizes known results on Cauchy problems in lattice cones and provides a new perspective on computational techniques for solving multidimensional recurrence relations. The work contributes to the ongoing development of algorithmic methods for discrete boundary value problems and has potential applications in combinatorics and digital signal processing.

This research was supported by the Russian Science Foundation (RSF) and the Krasnoyarsk Regional Science Foundation within the framework of the project “Methods of multidimensional difference equations and computer algebra in the theory of digital recursive filters”, project no. 25-21-20076.

Fuss-Catalan numbers and difference equations for under diagonal lattice paths

Sreelatha Chandragiri*†1

[1]*Sobolev Institute of Mathematics, 4 Acad. Koptyug avenue, 630090 Novosibirsk Russia, Russia*
†*E-mail: srilathasami66@gmail.com*

Abstract

In this paper, we give some definitions and basic properties of Fuss-Catalan numbers. Fuss-Catalan number is a family of generalized Catalan numbers. This family can be generalized by counting some restricted lattice paths that lie on or under the line $y = mx+c$ by using difference equations with non-constant coefficients. Also, we define this family in the language of generating functions. Finally, we give combinatorial proof of an identity $Ak(p, r) = Ak(m + 1, c + 1)$ where $Ak(p, r)$ are the Raney-numbers.

We concern tree enumeration based on difference equations by taking into consideration of restricted lattice path problems. Fuss-Catalan numbers can be thought of as higher dimensional Catalan numbers. We give some combinatorial interpretations different from the original Catalan numbers. For example, one interpretation of the Catalan number counts the number of binary trees with k edges whereas the Fuss-Catalan number counts the number of m-ary trees with k edges, where k and mare non-negative integers, such that $m \geq 1$.

Keywords: Fuss, Catalan number, Difference equation, Lattice path, Generating function

Inverse problems on dynamic equations on time scales

Syed Abbas*†1

1 *School of Mathematical, Statistical Sciences, Indian Institute of Technology Mandi, Himachal Pradesh, India*
† *E-mail: abbas@iitmandi.ac.in*

Abstract

In this talk, we will pose some inverse problems on a time scale domain and discuss the existence and uniqueness of solutions for these problems. More specifically, we will consider identification problems of determining the unknown functions, coefficients, matrices or some particular matrix entries. We will prove some results explicitly and discuss the challenges in other problems.

References

[1] Martin Bohner and Allan Peterson, *Dynamic equations on time scales : An introduction with applications*, Birkhäuser Boston, Inc., Boston, MA, 2001.

[2] Alexander M Denisov, *Elements of the theory of inverse problems*, Inverse and Ill-posed Problems Series, De Gruyter, Berlin, Boston, 2014.

[3] Sergey Igorevich Kabanikhin, *Definitions and examples of inverse and ill-posed problems*, Journal of Inverse and Ill-Posed Problems **16** (2008), no. 4, 317–357.

[4] Alfredo Lorenzi, *An introduction to identification problems via functional analysis*, De Gruyter, 2014.

[5] Ganesh Ashok Satpute and Syed Abbas, *Parameter identification for vector dynamic equations on arbitrary time scales*, Acta Appl. Math. **183** (2023), Paper No. 6, 19.

[6] Ganesh Ashok Satpute and Syed Abbas, *Inverse Coefficient Problems for Dynamic Equations on Time Scales*, Math. Methods Appl. Sci. **48** (2025), no. 9, 10114–10127.

[7] Ganesh Ashok Satpute, Marko Kostić, and Syed Abbas, *Inverse problems for a system of dynamic equations on time scales*, J Anal (2025).

A discrete prüfer framework for spectral analysis of difference equation

Kürşat Er*†1 and F. Ayça Çetinkaya*‡2

[1] *Mersin University, Department of Mathematics, 33343, Mersin, Türkiye*
[2] *Department of Mathematics, University of Tennessee at Chattanooga, 37403, Chattanooga, TN, USA*
† *E-mail: erkursat89@gmail.com*
‡ *E-mail: fatmaayca-cetinkaya@utc.edu*

Abstract

In this talk, we present a Prüfer-based approach for analyzing second-order Sturm–Liouville difference equations and estimating eigenvalues of associated boundary value problems. By reformulating the original system in terms of amplitude and phase variables, we derive a discrete Prüfer transformation that provides new insight into the oscillatory behavior of solutions. This formulation enables a qualitative study of zero distributions and extremal points, and leads to a rigorous proof of equivalence between the transformed and original systems. We also establish Lipschitz continuity of the phase equation, ensuring uniqueness and stability of the Prüfer system under mild conditions. Building on this foundation, we design a practical shooting algorithm for eigenvalue estimation, with initial bounds incorporated to enhance computational efficiency. The method offers a unified framework with both theoretical value and numerical utility for discrete spectral problems.

References

[1] M. Bohner and O. Došlý, *The discrete Prüfer transformation*, Proc. Amer. Math. Soc., **129** (2001), no. 9, 2715–2726.

[2] M. D. M. Abumosameh, *Python Code for Sturm–Liouville Eigenvalue Problems*, Ph.D. thesis, Carleton University, 2022.

Eigenvalue comparisons for a conformable boundary value problem with impulse

Nick Wintz*†[1]

[1] *Radford University, United States*
† *E-mail: njwintz@gmail.com*

Abstract

We consider a pair of linear conformable differential equations subject to Sturm–Liouvile boundary conditions and a single impulse. We construct a Green's function associated with these boundary conditions. We construct a Banach space and related cone for which compare our smallest possible eigenvalues related to our boundary value problem.

Bessel functions on time scales

Tom Cuchta[*†1]

[1] *Marshall University, United States*
[†] *E-mail: cuchta@marshall.edu*

Abstract

Discrete Bessel functions on the time scale $T = Z$ were introduced in 2017, and they have been used in a variety of applications including discrete wave equations and semidiscrete diffusion equations. We present a generalization of the discrete Bessel functions to time scales through power series. Various properties of these functions will be presented, including a discussion on how the delta- and nabla-Bessel functions defined in this way have different convergence properties.

Special Session 8

Multiscale nonlocal reaction-diffusion modeling of phenotypic plasticity and drug resistance in glioblastoma

Ming Yan[1] and Xiaoqiang Sun[*†1]

[1]*Sun Yat-Sen University, China*
[†]*E-mail: sunxq6@mail.sysu.edu.cn*

Abstract

Glioblastoma (GBM) is a highly malignant brain tumor characterized by significant heterogeneity in both the tumor mass and its tumor microenvironment (TME). This heterogeneity manifests through two dynamic processes: the phenotypic plasticity of tumor cells transitioning between proneural (PN) and mesenchymal (MES) states, and the continuous polarization spectrum of tumor-associated macrophages (TAMs) ranging from tumor-suppressive M1to tumor-promoting M2 phenotypes. Clinical and experimental studies demonstrate that phenotypic plasticity of GBM and TAMs is associated with heterogeneous tumor progression and differential immunotherapeutic responses. However, the interplay between tumor cells and TAMs underlying their phenotypic transitions is poorly understood. In this study, to investigate the intrinsic mechanisms of GBM phenotypic transitions during both natural progression and drug treatment, we developed a nonlocal reaction-diffusion equations-based multiscale spatiotemporal model, incorporating TME-mediated signaling feedback loop between tumor cells and TAMs. Through rigorous mathematical analysis, we proved the existence, uniqueness, boundedness, and non-negativity of solutions to the model and derived stability conditions for equilibrium states. For numerical computations, we developed a radially symmetric implicit-explicit scheme for the nonlocal system of partial integro-differential equations. The model shows excellent agreement with experimental observations across multiple validation metrics, confirming its biological plausibility and predictive capability. Through global sensitivity analysis and parameter stability analysis, we systematically identified and quantitatively characterized key parameters regulating tumor growth dynamics and treatment response. Finally, we evaluated the efficacy of combination therapy regimens and proposed novel strategic approaches for optimizing GBM treatment.

Keywords: Multiscale model, Nonlocal reaction, Diffusion equations, Phenotypic plasticity, Drug resistance, Glioblastoma, Combination therapy

Modeling caspase-1-mediated pyroptosis of the predominance for driving CD4+T cells death

Wei Wang*†1

[1] *Shandong University of Science and Technology, China*
† *E-mail: weiw10437@163.com*

Abstract

Caspase-1-mediated pyroptosis is the predominance for driving $CD4{+}T$ cells death. Dying infected $CD4 + T$ cells can release inflammatory signals which attract more uninfected $CD4{+}T$ cells to die. This talk is devoted to developing some diffusive mathematical model which can make useful contributions to understanding caspase-1-mediated pyroptosis by inflammatory cytokines $IL{-}1\beta$ released from infected cells in the within-host environment.

Modeling tumour heterogeneity of PD-L1 expression in tumour progression and adaptive therapy

Shizhao Ma*†[1]

[1]*School of Mathematical Sciences, Shanghai Jiao Tong University, China*
†*E-mail: shizhaoma@sjtu.edu.cn*

Abstract

In this talk, I will first introduce how tumor cells achieve immune evasion through a key immune checkpoint—PD-L1. Based on a general mathematical framework for modeling heterogeneous stem cell regeneration (Lei model), we established a comprehensive modeling and computational framework for estimating the dynamic changes in PD-L1 heterogeneity during cancer progression and treatment, and predicting the overall survival of patients, and further explored the adaptive therapy of administering anti-PD-L1 according to dynamic of PD-L1 state among cancer cells. This talk is based on some joint works with Jinzhi Lei and Xiulan Lai.

Dynamic analysis of a tumor-immune model with state feedback impulsive control for tumor treatment

Yi Zhao*†1

[1] *University of Science and Technology Beijing, China*
† *E-mail: zhaoyi7144@163.com*

Abstract

Cancer, the second leading cause of death in the world, has long been a serious problem that plagues scientists. Cancer evolves from malignant tumors within the human body. The immune system in the human body is constantly struggling with tumor cells, where the relationship between tumor cells and effector cells can be seen as a prey-predator relationship among different species. The Beddington-DeAngelis functional response can better reflect the interaction between the two types of cells. Based on the consideration of the combination of traditional therapy and cellular immunotherapy, a state feedback impulsive tumor-immune model was established for the treatment of tumor in patients with mild and severe symptoms, respectively. On this basis, the existence of homoclinic orbit and order-1 periodic solution of the impulsive model was studied, and the stability of the periodic solution was discussed.

Keywords: Tumor immune interaction, Beddington DeAngelis functional response, State feedback, Impulsive model, Homoclinic orbit, Order 1 periodic solution

Dynamics of HPV transmission: Impact of screening awareness and vaccination in a two-sex deterministic model

Yuxin Zhang*†[1]

[1] *College of Mathematical Sciences, Harbin Engineering University, China*
† *E-mail: xyz.jl@163.com*

Abstract

In this talk, we investigate a two-sex deterministic model for human papilloma virus (HPV) transmission dynamics, focusing on vaccination and screening awareness effects. The model incorporates gender-specific differences in vaccine efficacy and screening awareness levels. We show the basic reproduction number is a critical threshold determining HPV persistence or control. Numerical simulations reveal increased vaccination rates, especially among males, significantly reduce HPV transmission more effectively than screening awareness alone, highlighting the importance of targeted male vaccination and screening awareness interventions.

Special Session 9

Existence and nonexistence of stable patterns in semilinear nonlocal diffusion equations

FANG LI*†1

[1] *Sun Yat-sen University*
† *E-mail: lifang55@mail.sysu.edu.cn*

Abstract

In this talk, we consider the dynamics of semilinear nonlocal diffusion equations on bounded domains with no-flux boundary conditions, specifically focusing on the existence and stability of non-constant steady states, referred to as patterns. According to the results of Casten, Holland, and Matano regarding semilinear local diffusion equations, we know that stable patterns do not exist in convex domains, while they do emerge in dumbbell-shaped geometries, particularly when the kinetic term is bistable. We extend these findings to nonlocal diffusion analogs, demonstrating the absence of stable smooth patterns in both one-dimensional intervals and multidimensional balls. In addition, we construct discontinuous, asymptotically stable patterns when the kinetic term is bistable. Our results reveal a significant principle: large nonlocal diffusion tends to destabilize patterns, whereas weak nonlocal diffusion stabilizes them, especially in cases with bistable kinetic terms. Importantly, the geometry of the domain appears to play a less critical role in this process of stabilization. This is joint work with Xueli Bai and Xuefeng Wang.

On a Lotka-Volterra competitive patch model

PENG ZHOU*†1

[1]Department of Mathematics, Shanghai Normal University
†E-mail: pengzhou011@126.com

Abstract

A Lotka-Volterra patch model describing the competition between two aquatic species is investigated, where organisms are supposed to take both random and advective movements. The global dynamics is determined in different settings of parameters, where sufficient analysis on the principal eigenvalue is performed to determine the local dynamics of the boundary equilibria and a technical approach is developed to establish the nonexistence of any positive equilibrium for any number of patches. This work can be seen as a further development of a series of works on the corresponding space-continuous PDE model.

Asymptotic behavior of the principal eigenvalue and basic reproduction ratio for periodic patch models

LEI ZHANG*†1

[1] *Shaanxi Normal University*
† *E-mail: lzhang@snnu.edu.cn*

Abstract

In this talk, we focus on the asymptotic behavior of the principal eigenvalue and basic reproduction ratio associated with periodic population models in a patchy environment for small and large dispersal rates. We first present the limiting profile of the principal eigenvalue of an associated periodic eigenvalue problem as the dispersal rate goes to zero and infinity, respectively. We then discuss the asymptotic behavior of the basic reproduction ratio in the case of small and large dispersal rates. Finally, we apply these results to a logstic patch model with drift and a benthic-drift patch model.

Global dynamics on an age-structured SIS epidemic patch model

Hao Kang*†1

[1] *Tianjin University*
† *E-mail: haokang@tju.edu.cn*

Abstract

We consider an age-structured SIS epidemic patch model. We define the basic reproduction number, and further analyze the effects of the diffusion rate and age structure on it. More precisely, we investigate their asymptotic behavior for small/large diffusion rates and with/without age structure. Finally we study the global behavior of this age-structured SIS epidemic patch model.

Dispersal-induced growth in the time-periodic environments

Shuang Liu*†[1]

[1] *Beijing Institute of Technology*
† *E-mail: liushuangnqkg@bit.edu.cn*

Abstract

Biologically, we are curious about the following question: How does the movement of individuals influence the persistence of a single population? A surprising phenomenon is that of dispersal-induced growth, whereby the population would become extinct if isolated, but dispersal, at an appropriate rate, can induce the persistence of the population. In this talk, we will discuss this topic based on a time-periodic two-patch model with non-symmetric dispersal matrix and introduce the corresponding principal eigenvalue to explain such counter-intuitive effect of dispersal.

Parameter identification of age-structured epidemic models

Junyuan Yang*†1

[1] *Shanxi University*
† *E-mail: yjyang66@sxu.edu.cn*

Abstract

Parameter identifiability analysis aims to determine whether model parameters can be uniquely determined from observable outputs. This is a critical step in epidemiological forecasting and the design of effective control measures. In this talk, we systematically review the theoretical foundations and practical applications of parameter identifiability and propose a framework for analyzing structural model parameter identifiability. By integrating principal component analysis (PCA) and eigenvalue analysis techniques, we systematically construct a hierarchical structure for parameter identifiability in age-structured epidemic models. Using Monte Carlo simulations and approximate Bayesian estimation methods, we identify parameters that are actually unidentifiable in the models. This is a joint work with Ziyi Wu and Maia Martcheva.

Effect of saturated incidence in an SIS epidemic patch model

Huicong Li*†1

[1]*Sun Yat-sen University*
†*E-mail: lihuicong@mail.sysu.edu.cn*

Abstract

We are mainly concerned with an SIS epidemic model with saturated incidence and constant total population in a spatially heterogeneous patchy environment. Firstly, in terms of the basic reproduction number, threshold type global dynamics is obtained in some special scenarios via construction of Lyapunov functions. Secondly, existence, uniqueness and non-existence of endemic equilibrium are established in certain parameter ranges. To investigate the effect of the saturation coefficient in our model, we then analyze asymptotic profiles of endemic equilibrium as the motility rate of the susceptible and/or infected population becomes small, or the saturation parameter is sufficiently large. Finally, we perform qualitative analysis on the corresponding model with linear birth/death effect to seek the influence of the variation of total population. Our results here, together with those of other closely related epidemic models, indicate that individual movement, infection mechanism, spatial heterogeneity and varying total population all may play essential roles in the spreading and control of infectious diseases; in particular, we conclude that the saturation parameter $m > 0$ in our model tends to induce a smaller number of infected population, compared to the patch system with bilinear incidence where $m = 0$.

The bifurcation dynamics of reaction-diffusion systems defined on networks

Wei Gou*†1

1 *Complex Systems Research Center, Shanxi University*
† *E-mail: wgou@sxu.edu.cn*

Abstract

Bifurcation theory has long served as a powerful tool-akin to a delicate surgical scalpel-enabling the dissection of dynamical system behaviors. While the theory was well-established for general ordinary differential equations (ODEs), its application to reaction-diffusion systems on networks remained challenging due to their ultra-high dimensionality. To overcome this longstanding hurdle, we conducted a series of investigations aimed at establishing a general computational framework for deriving bifurcation normal forms in such systems. Our work systematically addressed steady-state bifurcation, Hopf bifurcation, Turing-Hopf bifurcation, and more general bifurcation scenarios. Through our analysis, we discovered that the intricate topological structure of underlying networks could induce novel bifurcation types, thereby enriching the dynamical landscape of networked systems. Furthermore, we explored how topological perturbations—even subtle ones—could trigger and modulate simple bifurcations, shedding light on the interplay between network architecture and dynamical behavior. These researches not only advance the theoretical understanding of bifurcations in high-dimensional networked systems but also provide practical tools for analyzing complex systems in biology, ecology, and engineering, where reaction-diffusion processes on networks play a crucial role.

Ideal free dispersal of a multiple competing model with time periodicity

Hua Zhang*†1

[1]Shanghai Jiao Tong University
†E-mail: zhua@sjtu.edu.cn

Abstract

In this talk, we investigate the evolution of dispersal, particular the evolutionary stability of dispersal strategies that are capable of producing ideal free distribution (IFD) through a time-periodic patch model, where N species have identical ecological dynamics but distinct dispersal strategies. We extend the concept of IFD, or joint IFD, previously established in temporally constant or temporally periodic but spatially continuous environments, to time-periodic patch environments and derive a sufficient and necessary condition for IFD to be feasible. Under this condition, we demonstrate two competitive advantages of ideal free dispersal strategies: when a subset of species adopts an ideal free dispersal strategy, the solution of the initial value problem must converges to an IFD; if a unique species combination achieves a joint IFD, then this subset will dominate and competitively exclude the other species. Furthermore, we show that ideal free dispersal strategies are the only evolutionarily stable strategies.

Asymptotic behavior of the principal eigenvalue and basic reproduction ratio for periodic patch

Shanshan Chen*†1

[1]*Harbin Institute of Technology, China*
†*E-mail: chenss@hit.edu.cn*

Abstract

In this paper, we investigate a two-species Lotka-Volterra competition patch model in a Y-shaped river network, where the two species are assumed to be identical except for their random and directed movements. We show that competition exclusion can occur under certain conditions, i.e., one of the semi-trivial equilibria is globally asymptotically stable. Specifically, if the random dispersal rates of the two species are equal, the species with a smaller drift rate will drive the other species to extinction, which suggests that smaller drift rates are favored.

Special Session 10

A new class of the dynamic viscoplastic frictional contact problem with adhesion

Furi Guo*†1 and Jinrong Wang[2]

[1]*Shanxi Datong University, China*
[2]*Guizhou University, China*
†*E-mail: dtdxgfr@163.com*

Abstract

In this paper, our main goal is to study a new mathematical model which describes the frictional contact between a foundation and a deformable body which is composed of viscoplastic materials and where the process is considered dynamic. The contact condition on the normal plane is modeled by a unilateral constraint condition for a version of normal velocity in which the memory effect and the adhesion are considered. On the tangential plane a frictional contact condition is governed by the Clarke subdifferential of a locally Lipschitz function, and the evolution of the bonding field is governed by an ordinary differential equation. We formulate this problem as coupled system that consists of two ordinary differential equations and a variational-hemivariational inequality. Then, the existence, uniqueness and continuous dependence of the solution on the data results concerning the abstract system are established. Finally, we use the abstract results to show the existence and uniqueness of the solution to the contact problem.

Turing instability for a discrete cooperative system

Shuyun Jiao*†1, Hongfeng Ren[2] and Tong Wang[1]

[1]Shanxi Normal University, China
[2]Pearl River College of South China Agricultural University, China
†E-mail: jiaosy1979@foxmail.com

Abstract

In the present work, we have studied the Turing instability and pattern formation in a class of discrete Lotka–Volterra cooperative system. We establish the conditions for Turing instability and identify the precise Turing bifurcation when the diffusion coefficient is used as a bifurcation parameter. Within Turing unstable regions, a variety of Turing patterns are explored via numerical simulations, encompassing lattice, nematode, auspicious cloud, spiral wave, polygon, and stripe patterns, as well as their combinations. The periodicity and complexity of these patterns are verified through bifurcation simulations, Lyapunov exponent analysis, trajectory or phase diagrams. These patterns demonstrate the same diagrams as that of competitive system.

Keywords: Cooperative system, Turing instability, Pattern formation

Existence and nonexistence of positive solutions for a class of fractional q-difference equation

Shugui Kang*†1, Yonghui Han[2] and Chen Fu[1]

[1]*Shanxi Datong University, Xingyun Street, Wenxing Road, Datong City, Shanxi Province, China*
[2]*Shanxi General Aviation Vocational and Technical College, No.1, Hangyuan Road, Tonghang Industrial Park, Yunzhou District, Datong City, Shanxi Province, China*
†*E-mail: dtkangshugui@126.com*

Abstract

In this paper, we consider the existence and nonexistence of positivesolutions for a class of fractional q-difference equation. We obtain sufficientconditions on existence of one solution, two positive solutions and nopositive solution by applying Guo-Krasnosel'skii fixed point theorem on cones.

Keywords: Fractional q, Differences, Boundary value problem, Positive solutions, Fixed, Point theorem

Existence and local stability of steady-state solutions for a discrete Nagumo equation with two neurons

Yanglan Ou[1], Li Jiayi[2], Jiawei Huang[2], Mingyao Wen[2] and Guang Zhang*†[2]

[1]*School of Pharmacy, Guangzhou Xinhua University, China*
[2]*Zhujiang College, South China Agricultural University, China*
†*E-mail: qd_gzhang@126.com*

Abstract

In this talk, a discrete Nagumo equation with two neurons will be invetigated, the comparison principle, the invariant intervals, the existence of nondegradable steady-state solutions, and the local stability of steady-state solutions will be established. There are some different dynamical behaviors between such equation and its corresponding continuous-time versions.

Keywords: Discrete Nagumo equation, Comparison principle, Invariant interval, Local stability

Efficient computation of eigenvalues and eigenvectors for discrete reaction-diffusion systems using block tridiagonal matrices

Ying Xiong[1] and Changjian Wu*†[1]

[1] *Zhujiang College, South China Agricultural University, China*
† *Email: wchjianyf@163.com*

Abstract

We address the computation of eigenvalues and eigenvectors of the Jacobian matrix at a fixed point for a discrete reaction-diffusion system. First, we transform the system into matrix form and derive the Jacobian matrix at the fixed point, then generalize it to a special class of block tridiagonal matrices. By utilizing conditions requiring simultaneous diagonalizability of the blocks, we obtain the Jordan canonical form for block tridiagonal matrices with complex entries and the real Jordan canonical form for block tridiagonal matrices with real entries. From these Jordan forms, we determine the characteristic polynomial. Notably, these results are also applicable to tridiagonal matrices. Finally, we apply these theorems to the discrete reaction-diffusion system, obtaining the real Jordan canonical form of the Jacobian matrix at the fixed point. This proves particularly useful for analyzing Turing instability in the discrete reaction-diffusion system.

Keywords: Discrete reaction, Diffusion system, Turing instability, Block tridiagonal matrices, Si multaneously diagonalizable, Jordan canonical form

The successive approximation method for the initial value problem of q-fractional differential equations

Caixia Guo*†1

[1]*Zhujiang College, South China Agricultural University, China*
†*E-mail: iris-gcx@163.com*

Abstract

The existence of solutions for the fractional q-difference initial value problem and the system are discussed by the continuous approximation method. Then the approximate solutions of the two types of special fractional order q-difference initial value problem(the q-fractional order pantograph equation and the systems of the q-fractional order pantograph equation) are obtained. And several properties of the approximate solutions of two types of special equations are mainly discussed and analyzed.

The Caputo-type fractional q-differential equation with affine periodic boundary value problem

Jianmin Guo*†1

[1] School of Mathematics and Statistics, Shanxi Datong University, Datong, 037009
† E-mail: dtdxguojianmin@163.com

Abstract

In this paper, the existence of solution to a class of Caputo-type fractional q-differential equation with affine periodic boundary value problems is studied. Firstly, the affine periodic boundary value problem of fractional q-differential equations is transformed into an integral problem, and then the existence of the solution of the boundary value problem is obtained by using the Leray-Schauder fixed point theorem and the Krasnoselskii fixed point theorem, and finally the validity of the two results is illustrated by two examples.

References

[1] Ernst T. Some results for q-functions of many variables. Rend. Padova, 2004..

[2] Ernst T. q-Bernoulli and q-Euler polynomials, an umbral Approach. International Journal of Difference Equations, 2006, 1(1): 31-80.

[3] Caputo S M .Free modes splitting and alterations of electrochemically polarizable media[J].Rendiconti Lincei, 1993, 4(2):89-98.

[4] Jackson, F. H. XI.-On q-Functions and a certain Difference Operator. Transactions of the Royal Society of Edinburgh, 1909, 46(02):253-281.

[5] Ernst T. The history of q-calculus and a new method. Mathematics, 2000.

[6] Agarwal,R. P.Certain fractional q-integrals and q-derivatives. Mathematical Proceedings of the Cambridge Philosophical Society, 1969, 66(02):365-370.

[7] LIU Yang. Existence of positive solutions for boundary value problem of nonlinear fractional q-difference equation. Applied Mathematics, 2013, 4(10): 1450-1454.

[8] Graef J, Kong L. Positive solutions for a class of higher order boundary value problems with fractional q-derivatives. Appl. Math. Comput., 2012, 218: 9682-9689.

[9] Ahmad B, Nieto J J. Anti-periodic fractional boundary value problems. Computers and Mathematics with Applications, 2011, 62(3): 1150-1156.

[10] Gao Shanshan; Wu Rui; Li Cuiying. The Existence and Uniqueness of Solution to Sequential Fractional Differential Equation with Affine Periodic Boundary Value Conditions. Symmetry, 2022, 14(7):1389-1389.

[11] LI Xinhui, HAN Zhenlai, SUN Surong. Anti-periodic boundary value problems for fractional q-difference equations. Journal of Applied Mathematics and Computing, 2016, 50(1 / 2): 243-257.

[12] Pathak R P , Somvanshi P S .On existence result of impulsive antiperiodic boundary value problem of fractional order q(1, 2). Acta Ciencia Indica, 2015, 41(4):331-346.

[13] Ahmad B, Tariboon J, Ntouyas S K, et al. Existence results for impulsive fractional q-difference equations with anti-periodic boundary conditions. Boundary Value Problems, 2016(1):1-5.

[14] Zuo, Mingyue, Xu, Hao. Existence Results for Impulsive Fractional q -Difference Equation with Antiperiodic Boundary Conditions. Journal of Function Spaces, 2018, 2018(147):1-9.

Special Session 11

Maximal tolerable fault set of robust mpc for passive fault tolerance with predefined performance

YILIN MI*†1, FENG XU‡2, XUEQIAN WANG[2] AND YU LIU[1]

[1] *Harbin Institute of Technology, China*
[2] *Tsinghua Shenzhen International Graduate School, Tsinghua University, China*
† *E-mail: miyilin@hit.edu.cn*
‡ *E-mail: xu.feng@sz.tsinghua.edu.cn*

Abstract

We address the problem of computing the maximal tolerable fault (MTF) set for a classical robust model predictive control (RMPC) scheme, given predefined performance specifications such as a feasible operating region and an ultimate bound. This set is defined such that, when faults occur within it, the controlled system always satisfies the specified performance requirements. We parameterize the MTF set as an ellipsoid and formulate convex optimization problems to compute its inner approximations under each performance requirement. These optimization problems can then be combined to obtain an integral MTF set. The resulting set also enables set-based analysis of the tolerable limits of RMPC under additive process and sensor faults, as well as multiplicative faults such as actuator faults.

Fast joint state and input moving horizon estimation for nonlinear systems

Songtao Du*†[1], Yiming Wan[1], Jun Xu[2] and Jinfeng Liu[3]

[1] *Huazhong University of Science and Technology, China*
[2] *Harbin Institute of Technology, China*
[3] *University of Alberta, Canada*
† *E-mail: s_t_du@hust.edu.cn*

Abstract

Although joint state and input estimation (JSIE) is well established for linear systems, it is not fully developed for nonlinear systems, especially when the unknown input is non-additive. This paper investigates the JSIE problem for nonlinear discrete-time systems with non-additive unknown input. We propose a joint state and input moving horizon estimation (JSI-MHE) method which repeatedly solves nonlinear least squares optimization online over a sliding window. The contributions of this study are threefold. Firstly, we establish the maximum a posterior formulation of the JSI-MHE problem. This enables a probabilistic interpretation of the arrival cost term that summarizes the information before each estimation window. Secondly, we propose a structure-exploiting Gauss-Newton (GN) iteration algorithm to solve the formulated JSI-MHE problem at a vastly reduced computational cost compared to using an off-the-shelf solver. Specifically, we develop a Schur complement based forward-backward recursion by exploiting the block pentadiagonal structure of the GN Hessian matrix. Thirdly, we extract covariance information of estimated state and input from the above GN iteration as a free by-product, which allows implicit arrival cost update with no computational overhead. To show well-posedness of the proposed algorithm, we discuss the condition that ensures positive definiteness of the GN Hessian and extracted covariance matrices. It is demonstrated by case study on two reactor systems that the proposed fast JSI-MHE algorithm significantly improves estimation accuracy, convergence rate, and robustness to poor initialization compared to two variants of unknown input extended Kalman filters, while being computationally efficient in real time.

Discrete-time control with application to coupled tank system: A ceiling function

Shyam Kamal*†1

[1] *Indian Institute of Technology [BHU Varanasi](IIT BHU)-Banaras Hindu University, Varanasi, Uttar Pradesh 221005, India.*
† *E-mail: shyamkamal.eee@iitbhu.ac.in*

Abstract

This paper introduces a novel convergence algorithm designed for discrete-time systems, with a specific emphasis on achieving fixed-step finite-time stability (FSFTS). By integrating the Lyapunov theorem with the ceiling function, the proposed method offers a distinct advantage over traditional finite-time and fixed-time stability approaches. While conventional methods lack the ability to precisely regulate convergence time, the presented algorithm ensures that the discrete-time system converges within a user-defined settling time. The key contribution of this work lies in the development of a discrete-time controller based on the proposed Lyapunov theorem for FSFTS. To demonstrate the practical viability of this theoretical framework, the study addresses the tracking problem in a coupled tank system. Experimental implementation of the discrete-time controller on this system confirms its effectiveness in achieving convergence within the predefined time frame.

Keywords: Lyapunov function, Finite, Time stability, Discrete, Time stability

Safe yet chaotic control of linear planar systems via parametric convex optimizatio

Songlin Yang*†1, Sorin Olaru[2] and René Lozi[3]

[1] *CentraleSupélec-L2S, CentraleSupélec-3, rue Joliot Curie, Plateau de Moulon, 91192 GIF-SUR-YVETTE Cedex, France*
[2] *Université Côte d'Azur (UniCA)-Laboratory J.A. Dieudonné-Parc Valrose, 28, avenue Valrose 06108 Nice Cedex 2, France*
† *E-mail: songlin.yang@centralesupelec.fr*

Abstract

We address the problem of computing the maximal tolerable fault (MTF) set for a classical robust model predictive control (RMPC) scheme, given predefined performance specifications such as a feasible operating region and an ultimate bound. This set is defined such that, when faults occur within it, the controlled system always satisfies the specified performance requirements. We parameterize the MTF set as an ellipsoid and formulate convex optimization problems to compute its inner approximations under each performance requirement. These optimization problems can then be combined to obtain an integral MTF set. The resulting set also enables set-based analysis of the tolerable limits of RMPC under additive process and sensor faults, as well as multiplicative faults such as actuator faults.

Keywords: Constrained control, Optimal control, Discrete, Time systems, Strange attractor

Special Session 12

Universal discrete model and data analysis of hormetic effects

Sanyi Tang*†1

[1] *Shanxi University, China*
† *E-mail: sytang@sxu.edu.cn*

Abstract

Under different external interventions, such as pesticide dosage usage and radiotherapy intensity, the effects or types of responses caused by pest populations and tumor cells vary, resulting in different types of dose-response curves, including monotonic, U-shaped, inverted U-shaped, and bell-shaped curves. To describe these different types of dose-response curves, researchers have proposed numerous complex models to fit various dose-response relationships. However, these models lack generality, making their application a significant challenge. To overcome these difficulties, we propose a universally applicable discrete Ricker model for hormetic effects. Based on this model, we consider the influence of factors such as lag effects, random effects, and spatial effects, and use it to fit the dose-response curves for various types found in pest control, traditional Chinese medicine formulations, and tumor immunotherapy. We find that the proposed discrete model provides a better fit for the dose-effect curves of drug toxicity effects.

Discrete switching population models and its application

Changcheng Xiang*†1

[1] *Hubei Minzu University, China*
† *E-mail: xcc7426681@126.com*

Abstract

The switching discrete host-parasitoid model concerning integrated pest management (IPM) has been proposed in present work, and the economic threshold (ET) is chosen to guide the switches. That is, if the density of host (pest) population increases and exceeds the ET, then the biological and chemical tactics are applied together. Those multiple control measures are suspended once the density of host falls below the ET. Firstly, the existence and stability of several types of equilibria of switched system have been discussed briefly, and two or three parameter-bifurcation diagrams reveal the regions of different types of equilibria including regular and virtual equilibria. Secondly, numerical bifurcation analyses show that the switched discrete system may have very complex dynamics including multiple attractors coexist and switched-like behavior among attractors. Finally, we address how the key parameters and initial values of both host and parasitoid populations affect the host outbreaks, switching frequencies or mean switching frequency, and consequently the relative biological implications with respect to pest control are discussed.

Modeling the spread of *Wolbachia* in mosquitoes with birth-pulse and insecticide impulsive spraying

Xianghong Zhang*†1

[1] *Southwest University, China*
† *E-mail: xhzhang0322@swu.edu.cn*

Abstract

Wolbachia as an innovative technique has been approved to inhibit the replication of dengue viruses in mosquitoes. We firstly build a birth-pulse model with sex structures to depict the nonlinear dynamics of mosquito population and then to investigate how *Wolbachia* can suppress or replace wild mosquitoes. The existence and stability of periodic solutions of the system are proved by analyzing its stroboscopic map. Under incomplete maternal transmission, there may be two pairs of bistable periodic solutions. Next, an impulsive differential model with four state variables are proposed to describe the spread of *Wolbachia* in mosquito population with considering the spray of chemical insecticide. We study the stability and permanence of periodic solutions for the system. The results indicate that the integrated use of *Wolbachia* and pulse spaying insecticide can not only significantly reduce the quantity of mosquitoes, but also help to achieving the strategy of mosquito eradication or increasing parameter regions for the success of the strategy of mosquito replacement.

Periodic switching and Beverton-Holt survival: A new perspective on mosquito suppression models

Zhongcai Zhu*†[1]

[1] *Guangxi Normal University, China*
† *E-mail: zczhu@gxnu.edu.cn*

Abstract

In recent decades, the sterile insect technique and incompatible insect technique have emerged as promising strategies for suppressing mosquitoes and controlling mosquito-borne diseases. To optimize these approaches and design cost-effective release strategies, mathematical models incorporating periodic switching and density-dependent survival probabilities have been developed to explore mosquito populations dynamics. These models often exhibit rich and intricate behavior. In this talk,we introduce a periodically switched mosquito population suppression model that employs the Beverton-Holt-type survival probability, replacing the classic Ricker-type survival probability that increases the complexity of model dynamics. Through theoretical analyses and numerical simulations, we identify that the dynamic complexity observed in such models primarily stems from the interactions between wild and sterile mosquitoes, rather than the specific form of the survival probability. To our knowledge, we are the first to uncover the source of dynamic complexity in such models. Our findings enhance the understanding of mosquito population dynamics and provide valuable insights into refining sterile mosquito release strategies to achieve more effective population suppression.

Rich dynamics of a predator-prey system with state-dependent impulsive controls switching between two means

Qianqian Zhang*†[1], Sanyi Tang[1] and Xingfu Zou[2]

[1] *Shaanxi Normal University, China*
[2] *Western University, Canada*
† *E-mail: zqianqian1225@163.com*

Abstract

This paper deals with the control of a prey species (as an unwanted species) in a predator-prey system. We consider a scenario where there are two control means available and they are applied in a state-dependent impulsive way, meaning that when the population of the harmful species is lower than a preset threshold, no control measure will be implemented; while when it reaches the threshold, the two control means will be used either in alternating order or random order. We formulate a general mathematical model for this scenario to evaluate the effect of such a control strategy by exploring the dynamics of this model. We define a one-dimensional map (Poincaré map) and by using the properties of this map, we derive sufficient conditions for the existence and global stability of an order-k periodic solution. By using the analogue of Poincaré criterion and bifurcation theory, we also establish sufficient conditions for a transcritical bifurcation near the predator-free periodic solution. Finally, we apply the results for the general model to two particular cases from two distinct fields: (I) integrated pest control and (II) tumour control with a comprehensive therapy. For (I), theoretical and numerical results show that the outbreak period of the pest is longer when two pesticides are applied randomly than when the alternating strategy is used. For (II), we find that the treatment frequency of drug rotation strategy is lower than that of no drug change strategy, and that the higher the control intensity, the lower the treatment frequency.

Special Session 13

Stochastic dynamics of complex gene regulatory networks

Chen Jia*†[1]

[1] *Beijing Computational Science Research Center, China*
† *E-mail: chenjia@csrc.ac.cn*

Abstract

Stochastic gene expression and gene regulation are among the most important and complex dynamic processes within the cell. Constructing appropriate mathematical models for these processes and thereby interpreting the associated experimental observations is an extremely challenging task. This line of research lies at the hotspot and frontier of applied mathematics and biomathematics. Drawing on the latest findings from my collaborators and myself, I will briefly introduce the stochastic models of gene expression dynamics and their underlying mathematical theory, with the hope of providing a small but stimulating contribution.

Multi-strain infectious disease transmission model and estimation of transmission advantage

Shi Zhao[*†1]

[1] *Tianjin Medical University, China*
[†] *E-mail: zhaoshi.cmsa@gmail.com*

Abstract

In this talk, we introduce a series of multi-strain infectious disease transmission models as well as the strain-specific reproduction numbers. We define the transmission advantage by using strain-specific reproduction numbers. A case study of COVID-19 outbreak in Nigeria will be introduced with the estimation of transmission advantage of SARS-CoV-2 genetic variants.

The research on purification of brucellosis in dairy farm based on stochastic model

Juan Zhang*†1

[1] *Shanxi University, China*
† *E-mail: zhangjuan1020@sxu.edu.cn*

Abstract

Brucellosis is one of the most prevalent zoonotic diseases in China, with the number of human cases increasing annually since 2019. In 2022, the Chinese government explicitly incorporated "purification and elimination" into the guidelines and requirements for animal epidemic prevention under the Animal Epidemic Prevention Law. Aligning with the national strategy that prioritizes farms as the starting point for brucellosis purification and gradually extends efforts to regional levels, we focus on dairy cattle farms as the research subject and establish a stochastic model by using continuous-time Markov Chains to analyze the spread of brucellosis. Based on the fitting results of monitored positive data from two scaled breeding farms, the model is applied to calculate the control reproduction number, the distributions of epidemic duration, and the final scale of infected individuals. The findings indicate that under purification measures (monthly quarantine and culling), the estimated purification period for these farms ranges from 4 to 8 months, with a final infection scale of 65% to 87%. Furthermore, considering the implications and practical significance of prevention and control measures, we adopt the final epidemic size as an evaluation criterion to conduct sensitivity analysis. This allows us to estimate the final infection rate based on varying breeding scales and initial infection proportions, thereby determining the threshold for the initial infection proportion across different breeding scales. This threshold serves as a critical indicator for deciding whether additional purification measures are necessary. The research findings of this study provide valuable theoretical support and practical guidance for the implementation of brucellosis purification efforts.

Statistical and dynamical modeling of tuberculosis transmission in Xinjiang

Lei Wang*†[1]

[1] *College of Medical Engineering and Technology, Xinjiang Medical University, China*
† *E-mail: wlei81@126.com*

Abstract

Tuberculosis is a chronic infectious disease caused by *Mycobacterium tuberculosis.* Xinjiang is one of the serious tuberculosis epidemic provinces in China and the situation of drug resistance is very severe.This study explores the epidemic trends and control measures of tuberculosis in Xinjiang through statistical modeling, dynamical modeling and qualitative analysis. Specifically: (1) Analyze the association between various meteorological factors (temperature, relative humidity, and wind speed etc.) and the incidence of tuberculosis in Xinjiang. (2) Analyze the impact of air pollutants ($PM_{2.5}$, PM_{10}, SO_2, NO_2, CO) on the prevalence of tuberculosis in Xinjiang. (3) Identify the contribution percentages of drug-sensitive and drug-resistant *Mycobacterium tuberculosis* infections among incident cases of tuberculosis, and estimate the relative contribution of primary and acquisition resistance attributable to drug resistant tuberculosis incidence in Xinjiang. (4) Estimate the population attributable risk percentage (PAR%) of drug resistant tuberculosis due to SO_2 exposure. (5) Analyze the differences between direct and indirect transmission in the spread of tuberculosis in Xinjiang.

The dynamics and density function of a stochastic SEIW brucellosis model with Ornstein-Uhlenbeck process

Buyu Wen*†1, Zhidong Teng[2], Linfei Nie[3], Zhiming Li[3] and Hong Cao[3]

[1] *Liaodong University, China*
[2] *Xinjiang Medical University, China*
[3] *Xinjiang University, China*
† *E-mail: wenbuyu1129@163.com*

Abstract

A stochastic SEIW brucellosis model with Ornstein-Uhlenbeck process is investigated. In the model, we considered some of the realities of brucellosis transmission, such as there exist incubation period and environmental pathogen contamination, and the brucellosis can also spread in incubation period and contaminated environment. The threshold values R_0^e and R_0^s for the stochastic extinction of disease and the existence of stationary distribution are established. Specifically, if $R_0^e < 1$ then brucellosis almost surely exponentially dies out, otherwise, if $R_0^s > 1$ then brucellosis are persistence in the mean. Furthermore, under the condition $R_0^s > 1$ the model also has at least one stationary distribution, ergodic property and a unique normal density function around a quasi-positive equilibrium. The specific expression of the marginal probability density function is obtained through the numerical simulation. In addition, the numerical simulation indicates that, in the real world, environmental interference may promote the spread of brucellosis.

Keywords: Stochastic SEIW brucellosis model, Ornstein, Uhlenbeck process, Stationary distribution, Density function, Extinction

Global competitive dynamics of *Aedes Aegypti* and *Aedes Albopictus*

Hongpeng Guo*†[1]

[1] *Guangzhou University, China*
† *E-mail: ghp8013@gzhu.edu.cn*

Abstract

Concerned is the global competitive dynamics of *Aedes aegypti* and *Aedes albopictus*. We develop a planar time-switched model that considers favorable-unfavorable seasonal shifts, the interspecific mating competition in the favorable season, and the overwintering capacity in the unfavorable season. During the favorable season, we find a sharp estimation of a separatrix that determines the fate of *Aedes aegypti* and *Aedes albopictus*. When the two species undergo seasonal transition, the separatrix disappears and the time switching results in the discontinuity of the vector field, which causes complex dynamical behaviors. By analyzing the associated Poincaré map, we obtain sufficient and/or necessary conditions for the existence and stability of periodic solutions. The results show that asymmetric mating interference and wintering ability in suitable seasons were important factors affecting the interaction dynamics of *Aedes aegypti* and *Aedes albopictus*. By understanding and potentially manipulating these interactions, it might be possible to effectively reduce the size of unwanted species.

Special Session 14

Extinction and quasi-stationary for discrete-time endemic SIS and SIR models

Jifa Jiang*†1

1 School of Mathematics and Statistics, Henan Normal University, China
† E-mail: xu.feng@sz.tsinghua.edu.cn

Abstract

This talk studies stochastic discrete-time endemic SIS and SIR models whose susceptible individuals escape infection probability is $(1+\beta I)^{-1}$ replacing the exponential one in [1], its advantage is that endemic fixed point can be solved out. It is first proved that the mean-field of stochastic discrete-time SIS model has dichotomy dynamics: if $R_0 \leq 1$, then the frequency of infected individuals asymptotically converges to zero; if $R_0 > 1$, then it admits a globally stable endemic fixed point I^*. The same result for the mean-field of stochastic discrete-time SIR model still holds but in the second alternative we must assume that the recovery rate γ is sufficiently small. It is well-known that the infection goes extinct in finite time with probability one in the stochastic models for all R_0 values. Using the large deviation method, we investigate its mean time to extinction and the quasi-stationary distributions. We then prove the following. As $R_0 > 1$, these mean times to extinction increase exponentially with the population size N and as $N \to \infty$, the quasi-stationary distributions weakly converge to a Dirac measure at the endemic fixed point (S^*, I^*). While $R_0 < 1$, the mean times to extinction are bounded above $(1-\alpha)^{-1}$, where $\alpha < 1$ is the geometric rate of decrease of the infection; as $N \to \infty$ the quasi-stationary distributions weakly converge to a Dirac measure at the disease-free fixed point $(1, 0)$.

References

[1] Schreiber, S. J., Huang Shuo, Jiang Jifa and Wang Hao, Extinction and Quasi-Stationary for Discrete-Time Endemic SIS and SIR Models, SIAM J. Appl. Math., Vol. 81, No. 5 (2021), pp. 2195–2217.

Positivity-preserving numerical algorithms for stochastic differential equations driven by fractional brownian motion

Xiaopeng Chen*†1

[1] *Shantou University, China*
† *E-mail: xpchen@stu.edu.cn*

Abstract

In fields such as finance and ecology, stochastic differential equations (SDEs) must strictly preserve the positivity of solutions to align with practical significance. However, traditional numerical methods exhibit limitations in addressing such issues. This paper focuses on the construction of positivity-preserving numerical algorithms for SDEs driven by fractional Brownian motion (fBm) with non-Markovian systems. Specifically, the proposed algorithm is applied to the Cox-Ingersoll-Ross (CIR) model driven by fBm, and numerical simulations are conducted to analyze its first moment and second moment.

Transient dynamics of seed dispersal mutualism system under demographic and environmental stochasticity

Zhipeng Qiu*†1

[1] *Department of Mathematics, Nanjing University of Science and Technology, Nanjing 210094, China*
† *E-mail: nustqzp@njust.edu.cn*

Abstract

This talk investigates the transient dynamics of a singular absorbed diffusion process modeling seed-dispersal mutualistic system under demographic and environmental stochasticity. By analyzing the sub-Markovian semigroup for the singular absorbed diffusion process, the existence, uniqueness and convergence of quasi-stationary distribution (QSD) are established. We comprehensively analyze the impact of stochasticity on the exit problem in seed-dispersal mutualism, extending beyond mean extinction time to examine extinction locations and the most probable exit path (MPEP), highlighting how noise source and intensity shape extinction dynamics. Using Freidlin-Wentzell (FW) large deviation theory, the exit problem is fully characterized for the system perturbed only by demographic stochasticity. For the system influenced by both stochastic effects, Onsager-Machlup (OM) functional theory is applied to derive the governing equations for MPEP. Our results reveal that environmental stochasticity significantly alters the order of magnitude of mean extinction time for mutualistic system, making extinction events analyzable within practical timeframes. Low-density plant and animal populations are strongly influenced by the noise source, with increasing noise intensity driving extinction locations toward specific points. By identifying extinction thresholds for MPEP, we reveal that the source of environmental stochasticity significantly effects their trends as noise intensity varies. This work highlights the role of stochasticity in shaping the transient dynamics of seed-dispersal system and provides a framework for predicting extinction in key ecosystems under demographic and environmental stochasticity. This work is done jointly with Yu Zhu, Tao Feng and Hao Wang.

Stochastic stability of monotone dynamical systems: The irreducible cooperative systems

Xi Sheng*†1

[1] *University of Science and Technology of China, China*
† *E-mail: sunx@mail.ustc.edu.cn*

Abstract

The current series of papers is concerned with stochastic stability of monotone dynamical systems by identifying the basic dynamical units that can survive in the presence of noise interference. In the first of the series, for the cooperative and irreducible systems, we will establish the stochastic stability of a dynamical order, that is, the zero-noise limit of stochastic perturbations will be concentrated on a simply ordered set consisting of Lyapunov stable equilibria. In particular, we utilize the Freidlin-Wentzell large deviation theory to gauge the rare probability in the vicinity of unordered chain-transitive invariant set on a nonmonotone manifold. We further apply our theoretic results to the stochastic stability of classical positive feedback systems by showing that the zero-noise limit is a convex combination of the Dirac measures on a finite number of asymptotically stable equilibria although such system may possess nontrivial periodic orbits.

Keywords: Stochastic SEIW brucellosis model, Ornstein, Uhlenbeck process, Stationary distribution, Density function, Extinction

Special Session 15

Epidemic waves for a diffusive two-group model with double nonlocal effects in lattice

CHUFEN WU*†1

[1]*Foshan University, China*
†*E-mail: wucfmath@fosu.edu.cn*

Abstract

We propose a lattice dynamical system that arises in a discrete diffusive two-group epidemic model with latency in a patchy environment. The model considers the SIS form and latency of the disease in group 1, while the SIR form without latency of the disease in group 2. The system includes double nonlocal effects, one effect is the nonlocal diffusion of individuals in isolated patches or niches, while the other effect is the distributed transmission delay representing the incubation of the disease. We demonstrate that there is a threshold value c^* that can determine the persistence or disappearance of the disease.

Keywords: Epidemic waves, A patchy environment, Nonmonotonicity, Nonlocal

Continuum limits of the generalized discrete nonlinear Schrödinger equation and gauge equivalent model

Liyuan Ma*†1

[1] *Zhejiang University of Technology, China*
† *E-mail: mly2016@zjut.edu.cn*

Abstract

We focus on an integrable higher-order discrete nonlinear Schrödinger (dNLS) equation, which yields the Lakshmanan-Porsezian-Daniel (LPD) equation in the continuum limit. We show that the rational solutions and breather solutions of the higher-order dNLS equation yield the counterparts of the integrable fourth-order NLS equation under proper continuous limits. Most importantly, we make integrable discretization for the linear spectrum problem of the integrable generalized Heisenberg ferromagnetic equation that is just the gauge equivalent structure of the LPD equation. Compatibility condition of the discrete linear spectrum problem leads to a discrete integrable generalized Heisenberg Ferromagnetic equation. It is revealed that the discrete linear spectrum problem of the discrete integrable generalized ferromagnetic equation converges to the corresponding spectrum problem of the integrable generalized Feisenberg spin model in the continuum limit.

Dynamics of impulsive nonlocal diffusive systems in shifting environments

Jiabing Wang*†1

[1] *China University of Geosciences (Wuhan), China*
† *E-mail: wangjb@cug.edu.cn*

Abstract

In order to investigate the effects of heterogeneous shifting environments on the dynamical behaviors for an invading species with long-distance free diffusion and birth pulse hybrid, we focus on an impulsive nonlocal diffusive system with spatiotemporal shifting heterogeneities, in which both scenarios of monotone and non-monotone reproductive terms are considered. The evolution of this hybrid system can be transformed into a discrete-time recursion composed of a continuous noncompact semi-flow and a discrete-time map. Firstly, we establish the spreading properties including the upward convergence and asymptotic annihilation of the recursive system. Secondly, we obtain the existence, uniqueness and nonexistence of forced waves of the saturation or extinction type in terms of the shifting speeds. Finally, we conduct numerical simulations to illustrate our analytical results and demonstrate rich dynamics driving by the interplay between the shifting speed of the habitat and the speed at which species spread in the optimal habitat.

Bistable pulsating waves for a stage-structured species in periodic discrete habitat

Jian Fang[1], Ming Xu[2] and Xiao Yu*†[3]

[1] *Harbin Institute of Technology, China*
[2] *Xi'an Technological University, China*
[3] *South China Normal University, China*
† *E-mail: 17621417550@163.com*

Abstract

In this talk, a nonlocal stage-structured population model in discrete periodic media is formulated. The existence of bistable pulsating wave as well as the uniqueness of wave speed is then established. In addition, the sign of wave speed is also investigated. This is a joint work with Prof. Jian Fang and Dr. Ming Xu.

New dynamical and algebraic results on the real Jacobian conjecture

Yuzhou Tian*†1

[1] *Jinan University, China*
† *E-mail: tianyuzhou2016@163.com*

Abstract

The real Jacobian conjecture was posed by Randall in 1983. This conjecture asserts that if $F = (f_1, \ldots, f_n) : \mathbb{R}^n \to \mathbb{R}^n$ is a polynomial map such that $\det DF(\mathbf{x}) \neq 0$ for all $\mathbf{x} \in \mathbb{R}^n$, then F is injective. We give some new dynamical conditions and algebraic conditions such that the real Jacobian conjecture holds. Our main tool is the qualitative theory of dynamical systems. These are joint works with Prof. YuLin Zhao, Prof. Changjian Liu and Assoc. Prof. Xiuli Cen, respective.

Special Session 16

Double-Hopf bifurcation of diffusive Holling-Tanner predator-prey model

Weihua Jiang*†1

1 *Harbin Institute of Technology, China*
† *E-mail: jiangwh@hit.edu.cn*

Abstract

Considering the effect of memory-based diffusion on the population dynamics of the predator-prey model with Holling-Tanner-type functional response function, we establish the conditions for the constant steady state to lose its stability through double-Hopf bifurcation. Further, by analyzing the normal form of the double-Hopf bifurcation, we prove that memory-based diffusion can lead to a bistable phenomenon, i.e., two stable spatially asynchronous periodic orbits with different wave numbers coexist in the model. It can also lead to a spatially inhomogeneous quasi-periodic orbit.

Keywords: Stochastic SEIW brucellosis model, Ornstein, Uhlenbeck process, Stationary distribution, Density function, Extinction

Eigenvalue problems of difference equation and its applications

Chunchenġ Wang*†1

[1] *Harbin Institute of Technology, China*
† *E-mail: wangchuncheng@hit.edu.cn*

Abstract

In this talk, I will present some results on the distribution of eigenvalues for a class of difference equation, and its applications in reaction diffusion equations with time delay involving in the diffusion terms and neutral differential equations, based on the joint work with Yanhui Fan.

Equivariant Hopf bifurcation and rotating waves/periodic oscillations of nonlocal reaction-diffusion delayed equations on 2D square domain

Xun Cao*†1

1 *Harbin Institute of Technology, China*
† *E-mail: caoxun@hit.edu.cn*

Abstract

In this talk, the rotating waves/standing waves/periodic oscillations arising from *equivariant Hopf bifurcation* on *2D square domain* will be discussed. Firstly, the third-order normal form of equivariant Hopf bifurcation for a nonlocal partial functional differential equations (PFDEs) on 2D square domain are established, which has three simpler cases depending on spatial mode. Correspondingly, one has three sets of concise formulas for computing coefficients of the simpler normal forms. Notably, for equivariant Hopf bifurcation with a multiplicity of 1 or 2, the developed explicit algorithm of computing the third-order normal form using original system parameters is both applicable. At last, by exploring pattern formations of a nonlocal Holling-Tanner model on 2D square domain near equivariant Hopf singularity with the aid of the established normal forms, stable *clockwise and anticlockwise rotating waves* with the shape of $\pm\phi_1 \cos(\omega_1 t+\theta_1)\cos\frac{x}{l} \pm \phi_2 \cos(\omega_1 t+\theta_2)\cos\frac{y}{l}$−like *arising from equivariant Hopf bifurcation with a multiplicity of 2*, and stable *periodic oscillations* with the shape of $\cos\omega_1 t$−like *arising from equivariant Hopf bifurcation with a multiplicity of 1*, are theoretically predicted and numerically displayed, while the standing waves with the shape of $\pm\phi_1\cos(\omega_1 t+\theta_1)\cos\frac{x}{l}$ or $\pm\phi_2\cos(\omega_1 t+\theta_2)\cos\frac{y}{l}$ are transient.

Lattice-based stochastic models motivate non-linear diffusion descriptions of memory based dispersal

Yifei Li*†1

[1] *Harbin Institute of Technology, China*
† *E-mail: yifeili@hit.edu.cn*

Abstract

The role of memory and cognition in the movement of individuals (e.g. animals) within a population, is thought to play an important role in population dispersal. In response, there has been increasing interest in incorporating spatial memory effects into classical partial differential equation (PDE) models of animal dispersal. However, the specific detail of the transport terms, such as diffusion and advection terms, that ought to be incorporated into PDE models to accurately reflect the memory effect remains unclear. In this talk, I will introduce a straightforward lattice-based model where the movement of individuals depends on both crowding effects and the historic distribution within the simulation. Through repeated stochastic simulation and numerical explorations of the mean-field PDE model, we show that the new PDE model accurately describes the expected behaviour of the stochastic model, and we also explore how memory effects impact population dispersal.

Propagation dynamics for an integro-difference system

YingLi Pan*†[1]

[1] *Renmin University of China, China*
† *E-mail: ylpan@ruc.edu.cn*

Abstract

In this lecture, we first review the background of the interested recursive system. Then we establish the theorems of spreading speed and traveling wavs for thin tailed dispersal kernels by overcoming the lack of monotonicity and compactness. Further, we prove a criterion for the non-monotonicity of traveling wave profiles in terms of an eigenvalue problem. Finally, we prove a sharp rate of accelerating propagation that arised from fat tailed dispersal kernels by estimating convolutions of kernel function.

Special Session 17

Dynamics of a diffusive Leslie-Gower model with both-density-dependent fear effect

Yanqiu Li*†[1]

[1]*Shanghai Institute of Technology, China*
†*E-mail: liyq@sit.edu.cn*

Abstract

The report mainly focuses on the steady-state bifurcation at the interior positive constant steady state of a diffusive Leslie-Gower model with both-density-dependent fear effect. Taking the growth rate of the predator as the bifurcation parameter and using Crandall-Rabinowitz bifurcation theorem, we discuss the local and the global steady-state bifurcation near the homogeneous steady state, and analyze the stability and the structure of the bifurcated spatially inhomogeneous steady-state solutions. The results show that either too high or too low growth rate or fear intensity is not conducive to the spatial pattern of the species, and a moderate value is beneficial to species survival.

Derivation of Field-Road models by effective boundary condition method

Siyu Liu*†1

1 *Northeastern University, China*
† *E-mail: Liusiyu@outlook.com*

Abstract

Berestycki et al.[JMB., 66, p743-766] established a Field-Road model, which could describe the spread of invasive species on a large field with a relatively narrow road. There were many interesting results about this model, but we are not aware of any rigorous derivation for their model. In this talk, we aim to seek a rigorous derivation of this model by the effective boundary condition method. Firstly, we start from a well defined problem, called full model, on a large field with a width $\delta > 0$ road, and then give some energy estimates. Finally we can derive the Field-Road model by sending $\delta > 0$. Moreover, we also derive a dynamic boundary condition (also Wentzell type condition) model from another full model with different transmission condition in a similar way.

The stability and Hopf bifurcation of solutions for fractional-order difference systems with time delays

Xiang Liu*†1

[1] *Hebei Normal University, Shijiazhuang, China*
† *E-mail: xliu@hebtu.edu.cn*

Abstract

The stability and Hopf bifurcation are important dynamical phenomenons in fractional-order difference systems. In order to grasp the internal law of these difference systems, it is interesting for us to study their stability and Hopf bifurcation behavior. By the study on Hopf bifurcation, we can find the critical value to control the stability region and the time of onset of Hopf bifurcation for such systems. In this talk, we have obtained some novel criteria about the stability and Hopf bifurcation of solutions for fractional-order difference systems using fractional-order difference operators theory, stability theory and bifurcation knowledge. Further, by designing a suitable feedback controller, the stability region and the time of appearance of Hopf bifurcation for the involved difference systems have been adjusted. Finally, an example is presented to illustrate one of the obtained results. The research ideas of this talk play a key role in studying the related fractional-order dynamical models in actual world.

Special Session 18

Using functional equations to study invariant measures

Janusz Morawiec*†1

[1] *University of Silesia, Poland*
† *E-mail: janusz.morawiec@us.edu.pl*

Abstract

We aim to demonstrate how invariant measures can be investigated through the study of solutions to functional equations. This approach is illustrated for iterated function systems as well as for single transformations.

Keywords: Invariant measures, Iterative functional equations, Probability distribution functions

Iterative roots and semiflows of mean-type mappings

Justyna Jarczyk*†[1]

[1] *Institute of Mathematics, University of Zielona Góra, Poland*
† *E-mail: j.jarczyk@im.uz.zgora.pl*

Abstract

In this talk I am going to recall somewhat forgotten results dealing with iterative roots and semiflows of mean-type mappings as well as to pose a number of related open questions. Here, given a real interval I, we study mean-type mappings $(M, N) : I \times I \to I \times I$, where $M : I \times I \to I$ and $N : I \times I \to I$ are means on I, that is the values $M(x, y)$ and $N(x, y)$ lie between $\min\{x, u\}$ and $\max\{x, u\}$ for all $x, y \in I$. As such pairs (M, N) are self-mappings of the square $I \times I$ we can examine their dynamics: iterates, existence of iterative roots, and embeddability in semiflows of mean-type mappings. The presented results originate in the below papers: [1], [2], [3].

References

[1] D. Głazowska, J. Jarczyk, W. Jarczyk, Square iterative roots of some mean-type mappings, J. Difference Equ. Appl. 24 (2018), 1729-1735.

[2] D. Głazowska, J. Jarczyk, W. Jarczyk, Embeddability of pairs of weighted quasiarithmetic means into a semiflow, Aequat. Math. 94 (2020), 679-687. Correction, Aequat. Math. 95 (2021), 599-600.

[3] W. Jarczyk, M. Matkowski, Embeddability of mean-type mappings in a continuous iteration semigroup, Nonlinear Anal. 72 (2010), 2580-2591.

Invariance of generalized quasiarithmetic means

Qian Zhang*†1

[1]*Southwest University of Science and Technology [Mianyang], China*
†*E-mail: zqianqian@snnu.edu.cn*

Abstract

In this report, we present a systematic investigation of Gauss's iteration of mean values and its associated invariant equation, exploring the fundamental relationship between these concepts. Our study focuses on the invariance of the arithmetic mean with respect to generalized quasiarithmetic means. Specifically, we establish new results characterizing this invariance for two distinct cases: (1) when the quasiarithmetic means are generated by identical measure functions, and (2) when they are generated by different measure functions. The analysis provides a unified framework for understanding the structural properties of mean value iterations and their invariant relations.

Data-driven discrete models for wstri spread in Nilaparvata lugens

HUICHAO YANG*†1

[1] *Guangzhou University, China*
† *E-mail: yhc0214@e.gzhu.edu.cn*

Abstract

Wolbachia, an intracellular bacterium, is well-known for inducing cytoplasmic incompatibility, which has become a promising and environmentally sustainable strategy for controlling pest populations. The strain *w*Stri, specifically identified in *Nilaparvata lugens* (brown planthopper), has shown potential for such biocontrol applications. In this study, we develop a comprehensive discrete mathematical model to analyze the dynamics of *w*Stri spread in a mixed population of *w*Stri-infected, *w*Lug-infected, and uninfected *Nilaparvata lugens* under both constant and periodically varying environmental conditions. Under a constant environment, the model identifies the critical threshold necessary for the successful establishment of *w*Stri within the population. Our analysis reveals that the model exhibits a strong Allee effect, where a population must exceed a certain critical density—the Allee threshold—for the *w*Stri strain to persist and spread. Below this threshold, the *w*Stri strain is likely to be eliminated, failing in pest control efforts. When the environment varies periodically, the model transforms into a non-autonomous periodic discrete model, introducing additional complexity. In this scenario, we derive sufficient conditions that ensure the composition of finitely many Allee maps continues to function as an Allee map. Furthermore, we prove that a unique periodic orbit exists within such a periodic environment. This orbit is characterized as unstable and acts as a threshold, determining whether *w*Stri will establish itself in the population or die out over time. The findings from this model provide critical insights into the conditions under which *w*Stri can be effectively used to control *Nilaparvata lugens*, particularly in environments that are not constant but fluctuate periodically. These insights have significant implications for the practical deployment of *Wolbachia*-based biocontrol methods in pest management strategies.

Special Session 19

Spatial homogeneity of stable solutions for almost periodic parabolic equations

Hongxu Li*†[1] and Zhi-An Wang[1]

[1] *Sichuan University, China*
† *E-mail: hoxuli@scu.edu.cn*

Abstract

In this talk, we consider the spatial homogeneity of linearly stable solutions of the almost periodic parabolic equation with variable diffusion under homogeneous Neumann boundary conditions. It is shown that any linearly stable almost automorphic (almost periodic) solution is spatially homogeneous under some conditions for the radially symmetric case with convex spatial domain. We also give the spatial homogeneity of linearly stable almost automorphic (almost periodic) solution when the nonlinearity is convex or concave without requiring convexity for the spatial domain. Moreover, we have the frequency module containment.

Bifurcation and pattern analysis in discrete predator-prey models

Xiaoling Han*†[1]

[1] *Northwest Normal University, China*
† *E-mail: hanxiaoling9@163.com*

Abstract

In this talk, we consisider the dynamic behavior of a class of spatiotemporally discrete predator-prey models with self-diffusion and cross-diffusion based on the coupled map lattice model. Theoretically, the conditions for the discrete system to undergo Flip bifurcation and Neimark-Sacker bifurcation are obtained, and the conditions for the discrete diffusion system to undergo Turing instability, as well as the conditions for the discrete diffusion system to exhibit pure Turing instability, Flip-Turing instability, and Neimark-Sacker-Turing instability are derived. The influence of self-diffusion and cross-diffusion parameters on the critical values of Turing bifurcation and the spatiotemporal dynamic regions of the discrete system is analyzed through numerical simulations. Finally, the impact of self-diffusion and cross-diffusion parameters on the spatial patterns of prey is analyzed. Numerical simulations demonstrate that a discrete diffusion predator-prey model with Holling-II type functional response function will undergo Turing instability under the influence of self-diffusion.

A study of chaotic behavior in some classic ecology models

Mingshu Peng*†[1]

[1] *Beijing Jiaotong University, China*
† *E-mail: mshpeng@bjtu.edu.cn*

Abstract

Chaos is the most important aspect of dynamical system research and is an objectively existing theoretical phenomenon. This report will reveal the process of the transformation from weak chaos to strong chaos in tent mapping and the interesting dynamics of the corresponding symbolic system, and thereby define the stability of infinitely long non-periodic orbits, which is one of the most important ideas of this research. Further, at the 2023 Differential Equations and Dynamical Systems Conference held by Professor Li Xianyi in Hangzhou, we proposed the probabilistic stability, and analyzed the existence and stability of infinitely long orbits, and explored rich dynamical behavior of Cantor sets. Two representative paths leading to chaos were discussed: 1. Gradual bifurcating phenomenon by periodic-doubling, such as the Logistic mapping, in this case, the chaos is in a weaker way; 2. The collision of the boundary by the torus rupture and boundary crisis with the stable manifold leads to chaos, in a stronger chaotic way, which exhibits the dynamics behavior like the shift mapping in the symbolic dynamical system, such as the Logistic mapping when the parameter is 4 or the classical tent mapping parameter with $\mu = 2$, or the delayed Logistic mapping like $x(k+1) = \mu x(k)(1 - x(k-1))$. Further, the path to chaos through the phenomenon of catastrophe change in non-smoothness systems is to be introduced: the representative mapping is the tent map.

Recent advances for investigations of rational difference equations

Xianyi Li*†[1]

[1] *Zhejiang University of Science and Technology, China*
† *E-mail: xianyili6590@163.com*

Abstract

In this talk, I will introduce some recent advances for investigations of rational difference equations, especially for some of their solutions for "Open Problems and Conjectures".

Global dynamics in a two-species competition patch model

Jie Liu*†1

1 *Harbin Institute of Technology, China*
† *E-mail: liujie9703@126.com*

Abstract

In this talk, we investigate a two stream species Lotka-Volterra competition patch model with the patches aligned along a line. The two species are supposed to be identical except for the diffusion rates. We treat one species as the resident species and the other one as a mutant/invading species and study whether the resident species can be invaded. We also obtain the global dynamics of the model for certain parameter ranges.

Hopf bifurcation of a spatially discrete reaction-diffusion equations with two components

Zunxian Li*†1

[1] *Tianjin University of Technology, China*
† *E-mail: lizunxian_hs@163.com*

Abstract

In this report, we first present spatially discrete reaction-diffusion equations with two components and the corresponding discrete boundary conditions. Secondly, we analyze the local stability of spatially homogeneous steady states by applying the matrix form of the decoupling method, and further derive the conditions for Hopf bifurcation. Furthermore, we discuss the properties of bifurcating periodic solutions using the Hopf bifurcation theorem. Finally, specific examples are provided to illustrate the application of the above theoretical analysis.

Special Session 20

Stability and bifurcation of a discrete predator-prey system with Allee effect and other food resource for the predator

Yuming Chen*†[1]

[1] *Department of Mathematics, Wilfrid Laurier University, Waterloo, ON N2L 3C5, Canada*
† *E-mail: ychen@wlu.ca*

Abstract

This talk concerns with a discrete predator-prey system with Allee effect and other food resources for the predator. Conditions on the existence and stability of fixed points are obtained. Then, with the center manifold theorem and bifurcation theory, fold bifurcation and flip bifurcation are analyzed. Numerical simulations are provided to illustrate the feasibility of the main results and the influence of Allee effect on the stability of the system. The study indicates that other food resources for the predator can enrich the dynamical behaviours of the system, including cascades of period-doubling bifurcation in orbits of period-2, 4, 8, and chaotic sets.

Bifurcation structures and invariant circle breakdown in the Chialvo neuron map

Yiwen Chen*†1 and Xianfeng Li[1]

[1]*Lanzhou Jiaotong University, China*
†*E-mail: yiwenchen9697@163.com*

Abstract

This work presents a detailed bifurcation study of the Chialvo neuron map. By deriving and analyzing the normal forms, the conditions for fold, flip, and Neimark-Sacker bifurcations are established, providing a theoretical basis for local stability changes. In the two-parameter plane, various strong resonance points, including 1:1, 1:2, 1:3, and 1:4 resonances, are detected. The parameter space further reveals shrimp-shaped periodic regions arranged according to a Farey tree structure, indicating a well-organized coexistence of periodic and quasiperiodic dynamics. In addition, critical curve are employed to examine the mechanisms responsible for the breakdown of invariant circles, clarifying how smooth closed curves disintegrate under parameter variation. These results show that resonance regions and critical boundaries determine the transition from periodic firing to chaotic oscillations in this neuron map.

Non-constant periodic solutions of the Ricker model with periodic parameters

Yu Gu*†1

1 *Xiangnan University, China*
† *E-mail: guyu@xnu.edu.cn*

Abstract

In this work, we will introduce some new results about the Ricker model

$$x_{n+1} = x_n \exp[r_n(1 - x_n)], \quad n = 0, 1, 2 \ldots, \qquad (*)$$

where $x_0 \geq 0$, and $\{r_n\}_{n=0}^{\infty}$ is a sequence of positive ω-periodic numbers.Some sufficient conditions on the existence of non-constant periodic solutions for equation $(*)$ are given. For the special case of period-two parameters, we show that $(*)$ has at most two non-constant 2-periodic solutions. We also obtain necessary and sufficient conditions for $(*)$ to have no, a unique, or exact two 2-periodic solutions, respectively. Examples are also given to illustrate our main results at last.

This is joint work with professors Fangfang Liao, Xiaoping Wang, Fulai Chen.

References

[1] B. Zheng and J. Yu, Existence and uniqueness of periodic orbits in a discrete model on *Wolbachia* infection frequency, Adv. Nonlinear Anal., 11(2022), 212-224.

[2] J. Yu and J. Li, Discrete-time models for interactive wild and sterile mosquitoes with general time steps, Math. Biosci., 346(2022),108797.

[3] J.Yu and B. Zheng, Modeling *Wolbachia* infection in mosquito population via discrete dynamical models, J.Differ.Equ. Appl., 25(11)(2019), 1549-1567.

[4] R. M. May, Biological populations obeying difference equations: stable points, stable cycles and chaos, J. Theo. Biol., 51 (1975), 511-524.

[5] R. M. May, G. F. Oster, Bifurcations and dynamic complexity in simple ecological models, Am. Nat. 110 (1976), 573-599.

[6] Q. Q. Zhang and Z. Zhou, Global attractivity of a nonautonomous discrete logistic model, Hokkaido Math. J., 29(2000), 37-44 .

[7] X. Wang, Y.Gu, J. Wang, F. Liao and B. Zheng, Periodic solutions and stability of a discrete mosquito population model with periodic parameters,Discrete Contin. Dyn.syst.29(2 024),4481-4491.

[8] Y.Gu, X. P.Wang, F. L. Chen, and F. F. Liao, Non-constant periodic solutions of the Ricker model with periodic parameters. J. Diff. Eqns. Appl., 30 (2024), 1842-1856.

[9] Z. Zhou and X. F. Zou, Stable Periodic Solutions in a Discrete Periodic Logistic Equation, Appl. Math. Lett., 16 (2003), 165-171.

Dynamics of discrete predator-prey model with prey harvesting

Xiaofeng Pang*†1

[1] *Guangzhou University, China*
† *E-mail: 2112215050@e.gzhu.edu.cn*

Abstract

In this talk, we propose a discrete Leslie-Gower predator-prey model with prey harvesting by the forward Euler method. Applying the center manifold theorem and bifurcation theory, we not only investigate the existence and stability of boundary and positive equilibria, but also derive bifurcations of the positive equilibrium. Moreover, we provide numerical simulations to illustrate the correctness of the obtained theoretical results and show that the discrete model has complex dynamic behaviors than the corresponding continuous model.

Ground state solutions of Nehari-Pankov type for a partial difference equation with periodic coefficients

Shaohong Wang*†1

[1] *Xi'an University of Science and Technology, China*
† *E-mail: shwang@xust.edu.cn*

Abstract

In this talk, we are concerned with the existence of ground state solutions of NehariPankov type for a partial difference equation with periodic coefficients, where associated energy functional is strongly indefinite. When the nonlinearity satisfies mixed super-quadratic and asymptotically quadratic at infinity, we establish some existence results of solutions by means of critical point theory, spectral theory of operators and non-Nehari manifold method.

Special Session 21

The stopping time and the spatial-temporal risk index in the epidemic model

Zhigui Lin*†1

[1] *Yangzhou University*
† *E-mail: zglin@yzu.edu.cn*

Abstract

In this talk, we first consider a SIVR (susceptible-infected-vaccinated-recovered) model which combines impulsive vaccination into the classical SIR model. The final size, the peak value and peak time are studied, then the critical times for a given infected number is discussed, and it can be used to define and estimate the stopping time. Finally, a diffusion SIS model is proposed to study the spatial spreading of virus. The free boundary is introduced to describe the spreading front of the infected interval. To check the effect of spatial heterogeneity and habitat characteristic on the spreading of the virus, the spatial-temporal risk index is given. Our results show that the virus will spread in the high-risk habitat; In a low-risk habitat, small initial infected habitat, small initial numbers of the virus and fast diffusion are beneficial for the virus to vanish. When the virus spreads in the whole habitat, the asymptotic spreading speed is given.

Dynamical analysis and tendency prediction of HCV hidden-transmission models with protection awareness

Fengying Wei*†1

[1]Fuzhou University
†E-mail: weifengying@fzu.edu.cn

Abstract

In this talk, we propose HCV models with protection awareness and hidden-transmission mechanism including exposed-acute-chronic phases for the population in the community. First of all, the basic reproduction number R_0 of the deterministic HCV model is derived by the next generation matrix method. The stochastic reproduction numbers such as R_0^s and R_0^e of the stochastic HCV model are respectively derived by constructing Lyapunov functions. Further, we show that the stochastic HCV model admits a unique global positive solution for any given positive initial values. We thus verify that the stochastic HCV model undergoes the persistence in the long run when $R_0^s > 1$ is valid. We also derive that the stochastic HCV model is extinct when $R_0^e < 1$ holds. Moreover, the relationships of the reproduction numbers R_0^s, R_0^e and R_0 are extensively investigated for the applications in the real circumstances. Consequently, the 2004-2022 monthly surveillance data are provided by the Fujian Provincial Center for Disease Control and Prevention, the HCV infection scales with single-measure, double-measure, triple-measure, and no-measure are extensively discussed. The numerical simulations show that both protection efficiency and conversion rate affect the HCV infection scale and reproduction numbers. The 2023-2035 tendency predictions reveal that the overall tendency of HCV in Fujian Province is consistent with the nation-wide elimination objective. This is a joint work with Zhen Jin and Xuerong Mao.

Time periodic traveling waves for multi-stable monotone semiflows and application to fractional diffusion equations

Weiwei Ding*†1

[1]*South China Normal University*
†*E-mail: dingweiwei@m.scnu.edu.cn*

This talk is concerned with the existence of traveling wave solutions for reaction-diffusion equations with fractional Laplacians in time-periodic environments. By developing a dynamical systems approach, we establish the existence of propagating terraces for equations with multistable nonlinearities, which consequently implies the existence of bistable traveling waves. Furthermore, we investigate whether traveling waves exist in the multistable case. We provide an affirmative answer to this question for a special type of nonlinearity in high-frequency oscillating environments, and determine the homogenized limit of the traveling waves as the period approaches 0.

Dynamics of discrete Ricker models on mosquito population suppression

Ruibin Jiang*†1

1 *Guangzhou University, Guangzhou, China*
† *E-mail: 1112315011@e.gzhu.edu.cn*

Abstract

Dengue fever is currently one of the most serious mosquito-borne infectious disease in the world. How to effectively prevent the outbreak of dengue fever has become a matter of significant public health concern. In this work, the cytoplasmic incompatibility induced by Wolbachia is assumed to be complete. Based on this assumption, we establish an extended discrete Ricker model with overlapping generations to investigate the suppression of mosquito population in the wild by adopting two different release strategies: the constant release strategy and the proportional release strategy. We prove the non-negativity, boundedness and stability of equilibrium points and finally find the release threshold, denoted as r_1^* and k_1^*, for the successful suppression in these two release strategies. In addition, we demonstrate that the model which adopts the constant release strategy respectively has a saddle node bifurcation when $r = r_1^*$ and a stable period-doubling bifurcation when $r = r_2^*$. While in the case of proportional release strategy, the model exhibits a transcritical bifurcation when $k = k_1^*$ and a stable period-doubling bifurcation when $k = k_2^*$. Finally, we substantiate the conclusions through numerical simulations.

Special Session 22

Estimation of transmission characteristics of SARS-CoV-2 infection and their implications for outbreak control

Zihao Guo*†[1]

[1] *The Jockey Club School of Public Health and Primary Care, The Chinese University of Hong Kong, China*
† *E-mail: guozihao9602@163.com*

Abstract

The COVID-19 epidemic caused by SARS-CoV-2 has given rise to tremendous public health burden worldwide. Timely and real-time assessment of the transmission risk is crucial to inform an efficient control strategy in response to emerging SARS-CoV-2 variants with immune escape capabilities. For an emerging infectious disease, knowledge of the epidemiological characteristics provides key insights into the susceptibility and infectivity associated with the transmission and determines the transmissibility of a circulating pathogen, which is often quantified by a single parameter – reproduction number. While the reproduction number is important it fails to reflect individual heterogeneities in transmission (i.e., superspreading event, in which an unusually large number of cases are infected by a single infectious person). Timely assessment of transmission heterogeneity and superspreading potential offers appropriate adjustments in control measures to effectively control the outbreaks. For the COVID-19 pandemic caused by SARS-CoV-2 and its variants, much effort has been taken to provide timely estimation of epidemiological parameters while the potential bias was often not taken into account. This study utilized case-based data from an in-depth outbreak investigation conducted in Hong Kong to demonstrate application of various statistical frameworks to obtain robust estimates of epidemiological parameters, informing real-time transmission risks and to provide insight into the effectiveness of control measures.

A two-stage sheep brucellosis transmission dynamic model in a patchy environment: Stability analysis and optimization of transportation scheme

Ting Kang*†1

1 *School of Mathematics and Statistics, Ningxia University, Yinchuan, Ningxia 750021, China*
† *E-mail: nxukangting@163.com*

Abstract

The cross-regional transport of sheep breaks geographical restrictions and expands the range of sheep, leading to the spread of brucellosis, one of the most widely spread zoonotic diseases transmitted by animals, and exposing more sheep and people to the threat of infection. To this end, this talk formulates a two-stage sheep brucellosis transmission dynamic model in a patchy environment based on the transmission characteristics of brucellosis. Firstly, the basic reproduction number is determined, and the global stabilities of disease-free, nontrivial boundary, and endemic equilibria are established, respectively. Subsequently, numerical simulations are applied to verify the correctness of the theoretical results. Finally, based on the recruitment rates of lambs and the basic reproduction number of two patches, the optimal transportation scheme for lambs and adult sheep is provided to reduce the risk of transmission of brucellosis.

Risk assessment of foot-and-mouth disease transmission based on network dynamics model

Xin Pei*†1

[1] *Taiyuan University of Technology, China*
†*E-mail: peixin1120@126.com*

Abstract

Based on data from the Xinjiang Animal Husbandry and Veterinary Big Data Platform, we constructed a livestock transportation network and analyzed its topological features, including degree distribution, centrality, and clustering coefficient to reveal important nodes and significant edges within the transportation network. In addition, within the livestock transportation network, the transmission mechanism of foot-and-mouth disease was introduced to establish a homogeneous mixing dynamic model within the livestock group and a discrete Markov process network model between the groups. The farms of four border cities and one central city were selected as sources of infection to simulate the transmission risk of foot-and-mouth disease in different regions of Xinjiang, including infection scale, infection areas, and infection risk. Finally, we applied spatiotemporal statistical methods to analyze the simulation results of the dynamics model, studying the clustering and outliers of foot-and-mouth disease in Xinjiang, as well as identifying hot outbreak areas, spatiotemporal aggregation characteristics, and the direction and speed of transmission.

Modeling and control of Brucellosis transmission dynamics in sheep with two-stage structure

Boqiang Cao*†[1,2]

[1]School of Mathematics and Statistics, Ningxia University, Yinchuan 750021, China
[2]Ningxia Basic Science Research Center of Mathematics, Yinchuan 750021, China
†E-mail: bcao@nxu.edu.cn

Abstract

Brucellosis is a zoonotic disease caused by bacteria of the genus Brucella, with lambs exhibiting a significantly lower infection rate compared to adult sheep. This talk analyzes the transmission mechanisms of sheep brucellosis and establishes a two-stage dynamic model. Furthermore, by introducing control variables and applying nonlinear adaptive control methods, the expressions for the control variables are designed, and the stability of the error model is analyzed. Additionally, considering scenarios where multiple farms share the same breeding environment, a multi-patch sheep brucellosis transmission model under common environmental influence is developed to investigate its adaptive control problem. Finally, numerical simulations are conducted to validate the effectiveness of the control strategies, and the control outcomes of the two types of models are compared.

Finite-time stability and optimal control of *Wolbachia*-driven mosquitoes based on stochastic differential equations

Wenjuan Guo*†[1]

[1] *Center for Applied Mathematics, Guangzhou University, Guangzhou, 510006, China*
† *E-mail: guowj@gzhu.edu.cn*

Abstract

It is well known that various environmental factors, such as temperature, rainfall and humidity, strongly influence the development and reproduction of mosquito populations and thus the transmission dynamics of mosquito-borne diseases. In this paper, a stochastic noise is introduced to describe the effects of environmental changes on mosquito population dynamics. Considering the waiting period of wild mosquitoes from mating to emergence, the finite-time stability of wild mosquitoes by releasing Wolbachia-infected mosquitoes was studied using a stochastic differential equation with time-varying delay. Finite-time stability describes the phenomenon that the bound of the state does not exceed a specified threshold at a fixed time interval. Sufficient conditions for the finite-time stability are obtained by employing the Lyapunov function and stochastic comparison theorem.And finally, the optimal control for the stochastic mosquito population model under proportional releases is researched.

Optimal control and cost-effectiveness of cystic echinococcosis dynamical model: A case study in Baiyin, Gansu Province

Qianqian Cui*†1

[1]*School of Mathematics and Statistics, Ningxia University, Yinchuan, Ningxia 750021, China*

†*E-mail: cmengqian@nxu.edu.cn*

Abstract

This talk presents an optimal control strategy and cost-effectiveness analysis for a dynamic model of cystic echinococcosis. A novel, time-varying model of echinococcosis was developed by integrating current prevention and control measures with the impact of health education. The model's basic reproduction number under constant control is calculated, and its global dynamic behavior is analyzed. Using optimal control theory, the optimal solution for the time-varying control model is derived. Model parameters were estimated based on actual data and control measures from Baiyin City, Gansu Province, China, and the resulting numerical fit proved satisfactory. Additionally, a numerical simulation of the optimal solution was performed for 24 proposed prevention and control measures, along with an analysis of cost-effectiveness. The results validated the effectiveness of Baiyin City's measures and indicated that implementing an integrated program of health education and treatment enhances the efficacy of prevention and control strategies. Findings further suggest that while health education or treatment alone can reduce echinococcosis transmission, a combined approach is more efficient and cost-effective. Specifically, a comprehensive prevention and control program–including human health education, safe disposal of infected livestock carcasses, preventing dogs from consuming infected organs, dog deworming, sheep immunization, and human treatment–is identified as the most cost-effective and effective strategy. A sensitivity analysis of key strategy parameters was also conducted. This study provides valuable theoretical support for the economically viable and effective prevention and control of cystic echinococcosis.

Accessibility of the three-year comprehensive prevention and control of brucellosis in Ningxia: A mathematical modeling study

Yu Zhao*†1

1 *School of Public Health, Ningxia Medical University, China*
† *E-mail: zhaoyu@nxmu.edu.cn*

Abstract

Brucellosis is a chronic zoonotic disease, and Ningxia is one of the high prevalence regions in China. To mitigate the spread of brucellosis, the government of Ningxia has implemented a comprehensive prevention and control plan (2022-2024). It is meaningful to quantitatively evaluate the accessibility of this strategy. Based on the transmission characteristics of brucellosis in Ningxia, we propose a dynamical model of sheep-human-environment, which coupling with the stage structure of sheep and indirect environmental transmission. We first calculate the basic reproduction number R0 and use the model to fit the data of human brucellosis. Then, three widely applied control strategies of brucellosis in Ningxia, that is, slaughtering of sicked sheep, health education to high risk practitioners, and immunization of adult sheep, are evaluated. The basic reproduction number is calculated as $R_0 = 1.47$, indicating that human brucellosis will persist. The model has a good alignment with the human brucellosis data. The quantitative accessibility evaluation results show that current brucellosis control strategy may not reach the goal on time. "Ningxia Brucellosis Prevention and Control Special Three-Year Action Implementation Plan (2022-2024)" will be achieved in 2024 when increasing slaughtering rate by 30% , increasing health education to reduce h to 50% , and an increase of immunization rate of adult sheep by 40%. The results demonstrate that the comprehensive control measures are the most effective for brucellosis control, and it is necessary to further strengthen the multi-sectoral joint mechanism and adopt integrated measures to prevention and control brucellosis. These results can provide a reliable quantitative basis for further optimizing the prevention and control strategy of brucellosis in Ningxia.

Precise control of Brucellosis in sheep: Dynamic modeling and analysis based on gender and size

Jun Yang*†1

[1]*School of Mathematics and Statistics, Ningxia University, Yinchuan, Ningxia 750021, China*
†*E-mail: 12024130664@stu.nxu.edu.cn*

Abstract

Brucellosis is a zoonotic disease caused by bacteria of the genus Brucella, which has extremely strong transmission capacity and high environmental tolerance. It not only poses a serious threat to public health, but also causes huge economic losses to the livestock. Medical research has found that there exist significant differences in the probability of brucellosis infection among sheep groups of different ages and genders in the same environment. Based on this phenomenon, this talk constructs an age-gender structured sheep-environment coupled transmission dynamical model. Firstly, we calculate the basic reproduction number of the model and analyze the local and global stability of the disease-free equilibrium point and the endemic equilibrium point. Then, three control measures, namely vaccination, isolation and environmental disinfection, are introduced. The target reproduction numbers corresponding to these three measures are calculated respectively, and their effects in controlling brucellosis are compared. The numerical simulation results show that the control effects of these three measures are different. Specifically, when the basic regeneration number $\mathcal{R}_0$ is less than 1, the control effect of environmental disinfection is the most significant. When $\mathcal{R}_0$ is greater than 1, vaccination can achieve the best control effect. Finally, by analyzing and comparing the costs of the two vaccination methods, we found that the cost of vaccination by gender is lower than that by age.

Special Session 23

The application of discrete dynamical systems in tuberculosis

Hui Cao*†1 and Yicang Zhou2

1 *Shaanxi University of Science and Technology, China*
2 *Xi'an Jiaotong University, China*
† *E-mail: caohui@sust.edu.cn*

Abstract

Based on the transmission mechanism of tuberculosis, this talk introduces several types of discrete-time infectious disease dynamics models. Main exploration Exogenous reinfection was discussed, different treatment methods were given to the latent individuals, age and age of infection, as well as the cyclical environment The influence of other factors on the dynamic behavior of discrete infectious disease models.

A Filippov model describing the effect of social distancing in controlling infectious diseases

Aili Wang*†[1]

[1] *Xi'an University of Technology (XUT), China*
† *E-mail: wangal@xaut.edu.cn*

Abstract

The spread of any infectious disease depends critically on contact between susceptible and infectious individuals. Social distancing is now a familiar strategy for managing disease outbreaks, but it is important to understand the interaction between disease dynamics and social behaviour. We distinguished the fully susceptibles from the social-distancing susceptibles and proposed a Filippov epidemic model to study the effect of social distancing on the spread and control of infectious diseases. The threshold policy is defined as follows: once the number of infected individuals exceeds the threshold value, social-distancing susceptibles take more stringent social-distancing practices, resulting in a decreasing infection rate. We found that one, two or three attractors coexist for the system as the threshold value and control parameters vary.

In particular, bistability of the regular endemic equilibrium and the disease-free equilibrium occurs for the system; multistability of the regular endemic equilibrium, pseudo-equilibrium and the disease-free equilibrium occurs for the system, revealing the vital role of the threshold value and the initial conditions. Discontinuity-induced bifurcations, including boundary node, focus and saddle-node bifurcations, occur for the proposed model, which reveals that a small change in threshold values would significantly affect the outcome. Our findings indicate that for a proper threshold value, if the initial number of infected individuals is sufficiently small, the number of infections can be ruled out or contained at the previously given value; otherwise, it may stabilize at a relatively high level.

Modeling and analysis of the transmission dynamics in metapopulation networks incorporating individual contact heterogeneity

Shanshan Feng*†1

[1]*North University of China (NUC), Taiyuan, Shanxi, China*
†*E-mail: fengshanshan@nuc.edu.cn*

Abstract

In studies of traditional metapopulation networks, researchers typically assumed that individuals within subpopulations were well-mixed, ignoring individual contact heterogeneity. However, in real life, individual contact exhibits high levels of heterogeneity. To address this, we build an SIS model that couples individual contact heterogeneity on a metapopulation network, which is characterized by constructing dynamic subnetworks within subpopulations. Theoretically, the basic reproduction number is obtained, the existence and uniqueness of equilibria, as well as their global stability, are proved. Through numerical simulations, theoretical results are validated. In addition, numerical simulations also indicate that individual contact heterogeneity is positively correlated with the basic reproduction number, and its effect on the number of infectious individuals is two-sided, which depends on infection rate and the degree of subpopulations. The research findings provide a scientific theoretical basis for the prevention and control of infectious diseases, thereby optimizing prevention and control strategies and achieving efficient resource utilization.

The critical community size on network

Hong Yang*†1

[1] *Taiyuan Normal University, China*
† *E-mail: yanghong_9002@163.com*

Abstract

A heterogeneous mean-field SIRS infectious disease stochastic dynamic model with population size is established, the main period of the model and the average extinction time of the disease are obtained, and the critical community size threshold of the model is obtained by using the relationship between the main period and average extinction time. The results show that: when average degrees of networks are equal but degree distributions differ, or vice versa, the thresholds for critical community size vary significantly. Population size is an important parameter affecting the extinction of stochastic infectious diseases, and this result can provide a theoretical basis for the control and eradication of infectious diseases.

Modeling and data analysis of emerging infectious disease transmission using stochastic difference equations

Sha He*†1

[1] *Shaanxi Normal University, China*
† *E-mail: hesha@snnu.edu.cn*

Abstract

This study develops stochastic difference equation models to characterize COVID-19 transmission dynamics under random effects. First, we establish a discrete-time stochastic epidemic model using binomial distributions, with parameters estimated from reported data. Next, we construct an improved model that incorporates asymptomatic transmission and imported cases to quantify resurgence risk and evaluate containment measures. An analysis of over 100 Chinese outbreaks reveals small-scale clustered transmission patterns under non-pharmaceutical interventions (NPIs). To investigate how stochastic factors influence the containment process, we derive a stochastic difference equation for newly reported cases based on the stochastic SIR framework and introduce the Stochastic Control Reproduction Number (SCRN). We further estimate the SCRN through Bayesian change-point analysis, and demonstrate via first-passage time theory that controlled randomness can accelerate epidemic containment.

Special Session 24

Nonuniform dichotomy spectrum and reducibility for nonautonomous difference equations

JIFENG CHU*†1

1 Hangzhou Normal University
† E-mail: jifengchu@126.com

Abstract

For nonautonomous linear difference equations, we introduce the notion of the so-called nonuniform dichotomy spectrum and prove a spectral theorem. As an application of the spectral theorem, we prove a reducibility result.

References

[1] J. Chu, F. Liao, S. Siegmund, Y. Xia, W. Zhang, Nonuniform dichotomy spectrum and reducibility for nonautonomous equations, Bull. Sci. Math. 139 (2015), 538-557.

[2] R. Sacker, G. Sell, Existence of dichotomies and invariant splitting for linear differential systems I [II, III], J. Differential Equations, 15 (1974), 429-458 [22(1976), 478-496, 497-522].

[3] R. J. Sacker, G. R. Sell, A spectral theory for linear differential systems, J. Differential Equations, 27 (1978), 320-358.

[4] S. Siegmund, Dichotomy spectrum for nonautonomous differential equations, J. Dynam. Differential Equations, 14 (2002), 243-258.

[5] S. Siegmund, Reducibility of nonautonomous linear differential equations, J. London Math. Soc. 65 (2002), 397-410.

Index of equilibrium point for planar piecewise smooth vector fields: Theory and applications

Shimin Li*†1

1 *Hangzhou Normal University*
† *E-mail: lishimin@hznu.edu.cn*

Abstract

The topological index, or simply the index, of an equilibrium point of a vector fields is an integer which saves important information about the local phase portrait of the equilibrium. There are mainly two ways to calculate the index of an isolated equilibrium point of a smooth vector fields. First Poincare and Bendixson proved that the index of an equilibrium point can be obtained from the number of hyperbolic and elliptic sectors that there are in a neighborhood of the equilibrium point, which is known as Poincare-Bendixson formula for the topological index of an equilibrium point. Second several works contributed to the algebraic method of Cauchy's index for computing the index of an equilibrium point. In this talk we generalize the Poincare-Bendixson formula and Cauchy's index to calculate the index of equilibrium point for planar piecewise smooth vector fields. This is a joint work with Changjian Liu, Jaume Llibre and Yulin Zhao.

References

[1] Z. Zhang, T. Ding, W. Huang, Z. Dong, Qualitative theory of differential equation, Translated of Mathematical Monographs, Providence, Rhode Island (1992).

[2] F. Dumortier, J. Llibre, J. Artés, Qualitative theory of planar differential systems, Springer, NewYork (2006).

[3] S. Li, C. Liu and J. Llibre, On the Poincaré index formula for planar piecewise smooth vector fields, J. Nonlinear Sci. 33 (2023) 118.

[4] S. Li, J. Llibre, Y. Zhao, On the index of an isolated equilibrium point for piecewise polynomial differential systems, J. Math. Anal. Appl. 551 (2025) 129651.

Bifurcation of limit cycles in a class of piecewise smooth generalized Abel equations with two asymmetric zones

Haihua Liang*†1

[1]*School of Mathematics and Systems Science, Guangdong Polytechnic Normal University*
†*E-mail: lianghhgdin@126.com*

Abstract

This paper studies the number of limit cycles, known as the Smale-Pugh problem, for the generalized Abel equation

$$\frac{dx}{d\theta} = A(\theta)x^p + B(\theta)x^q,$$

where A and B are are piecewise trigonometrical polynomials of degree m with two zones $0 \leq \theta < \theta_1$ and $\theta_1 \leq \theta \leq 2\pi$. By means of the first and second order analysis using the Melnikov theory and applying the new Chebyshev criterion that established by Huang, Liang and Zhang (J. Differential Equations, 2023), we estimate the maximum number of positive and negative limit cycles that such equations can have, and reveal how this maximum number, denoted by $H_{\theta_1}(m)$, is affected by the location of the separation line $\theta = \theta_1$. For the equation of classical Abel type, our result not only includes the estimates provided in the recent paper (Huang et al., SIAM J. Appl. Dyn. Syst., 2020), i.e., $H_{2\pi}(m) \geq 4m - 2$ for $\theta_1 = 2\pi$, but also shows that the equation in the discontinuous case can possess more than two times as many limit cycles as in the continuous case. More accurately, $H_{\pi}(m) \geq 8m + 2$ and $H_{\theta_1}(m) \geq 14m - 6$ for $\theta_1 \in (0, \pi) \cup (\pi, 2\pi)$.

Existence of large amplitude irrotational equatorial waves with flow force function

Xun Wang*†1

1 *University of Shanghai for Science and Technology*
† *E-mail: wangxun39@126.com*

Abstract

Based on a modified height function, we consider two-dimensional steady periodic equatorial water waves. In particular, we study the irrotational flows and deduce an equivalent system using the flow force function. We further demonstrate the existence of large-amplitude equatorial water waves by using the global bifurcation theory.

References

[1] B. Basu, A flow force reformulation for steady irrotational water waves, J. Differential Equations, 268 (2020), 7417-7452.

[2] B. Basu, C.-I. Martin, Capillary-gravity water waves: modified flow force formulation, J. Differential Equations, 269 (2020), 11231-11251.

[3] J. Chu, Q. Ding, Y. Yang, Steady periodic waves and formal stability for fixeddepth rotational equatorial flows, J. Differential Equations, 269 (2020), 4192- 4214.

[4] J. Chu, X. Wang, L. Wang, Z. Zhang, A flow force reformulation of steady periodic fixed-depth irrotational equatorial flows, J. Differential Equations, 292 (2021), 220-246.

[5] A. Constantin, W. Strauss, Exact steady periodic water waves with vorticity, Comm. Pure Appl. Math. 57 (2004), 481-527.

A new Chebyshev criterion and its application to differential systems

Jianfeng Huang*†1

1 Jinan University
† E-mail: thuangjf@jnu.edu.cn

Abstract

In this talk we will introduce a Chebyshev criterion for a certain family of integrals. By means of this criterion we establish several new Chebyshev families, which can help us to answer the problem proposed by Gasull et al. in 2015. The applications of these families to other differential systems also show that our approach is simpler and in a unified way to handle some kinds of differential systems for estimating the number of limit cycles bifurcating from period annulus.

Special Session 25

Periodic dynamics of a general switching dynamical system and two illustrating examples

Qian Ding*†1

[1] *Hunan City University*
† *E-mail: dqian@hncu.edu.cn*

Abstract

Seasonal changes and cyclical human activities (such as periodic fishing bans, *Wolbachia*-based mosquito population control, and school term breaks) have significant impacts on population dynamics. We propose a general switching dynamical model to describe these periodic changes. The existence, uniqueness, and stability of positive periodic solutions are thoroughly investigated and the results are stated in terms of an introduced threshold value. To demonstrate their practicability, the obtained results are applied to two biological situations.

References

[1] S. Hsu and X. Zhao, A Lotka-Volterra competition model with seasonal succession. J. Math. Biol., 64 (2012), 109-130.

[2] Y. Tang, D. Xiao, W. Zhang and D. Zhu, Dynamics of epidemic models with asymptomatic infection and seasonal succession, Math. Biosci. Eng., 14 (2017), 1407-1424.

[3] J. Yu and J. Li, Global asymptotic stability in an interactive wild and sterile mosquito model, J. Differ. Equ., 269 (2020), 6193-6215

[4] Y. Liu, J. Yu and J. Li, Global dynamics of a competitive system with seasonal succession and different harvesting strategies. J. Differ. Equ., 382 (2024) 211-245.

[5] X. Pan, H. Shu, L. Wang, X. Wang and J. Yu, On the periodic solutions of switching scalar dynamical systems, J. Differ. Equ., 415 (2025), 365-382.

Comparative studies of *Wolbachia* spread dynamics in mosquito populations via discrete models

Deyu Kong*†[1,2], Jianshe Yu[2] and Bo Zheng[1,2]

[1]*School of Mathematics and Information Science, Guangzhou University, China*
[2]*Center for Applied Mathematics, Guangzhou University, China*
†*E-mail: kongdeyu@e.gzhu.edu.cn*

Abstract

Depicting *Wolbachia* spread dynamics in mosquito populations using discrete dynamical models has a long history since 1959. All these models put the fitness cost to infected females alone while ignoring the fitness cost of Wolbachia infection to infected males. Based on the experimental observation that *Wolbachia* infection in males results in shorter longevity and decreased male fertility than uninfected males, we introduce a new discrete model to take the fitness cost of infected males into consideration. Comparative studies, focusing on the establishment threshold and the polymorphic state of Wolbachia, are carried out based on the characteristics of the *Wolbachia* strains. These results are important for the management and control of mosquito-borne diseases, especially when using *Wolbachia* as a biological control agent.

A discrete dynamical model on *Wolbachia*-infection frequency with periodic releases

Yantao Shi*†1

[1] *Foshan University*
† *E-mail: ytshi@fosu.edu.cn*

Abstract

Releasing *Wolbachia*-infected mosquitoes is an effective biological control method to impede the transmission of dengue. This talk introduces a discrete model with an impulsive and periodic release strategy to characterize *Wolbachia*-infection frequency. The model covers all relevant existing discrete models proposed and investigated by Caspari, Fine and others since 1959. We obtain sufficient conditions for the existence of T-periodic solutions, and numerical examples indicate that there exist at most two periodic solutions, which deserve further investigation. Its validity is theoretically demonstrated for the simplest case of 2-periodic releases. With incomplete cytoplasmic incompatibility and imperfect maternal transmission, our results demonstrate that periodic and impulsive releases can lead to the existence of exactly two periodic solutions, resulting in bistable dynamics.

References

[1] E. Caspari and G. Watson, On the evolutionary importance of cytoplasmic sterility in mosquitoes, Evolution, 1959, 13: 568-570.

[2] P.E.M. Fine, On the dynamics of symbiote-dependent cytoplasmic incompatibility in *Culicine* mosquitoes, J. Invertebr. Pathol., 1978, 31(1): 10-18.

[3] A.A. Hoffmann, B.L. Montgomery, J. Popovici, I. et al, Successful establishment of *Wolbachia* in *Aedes* populations to suppress dengue transmission, Nature, 2011, 476: 454-457.

[4] B. Zheng, J. Li, and J. Yu, One discrete dynamical model on *Wolbachia*-infection frequency in mosquito populations, Sci. China Math., 2022, 65(8): 1749-1764.

[5] B. Zheng and J. Yu, Existence and uniqueness of periodic orbits in a discrete model on *Wolbachia*-infection frequency, Adv. Nonlinear Anal., 2022, 11(1): 212-224.

A stochastic mosquito population suppression model based on incomplete cytoplasmic incompatibility and time switching

Rong Yan*†1

1 *School of Mathematics Science, Shanxi Normal University, Taiyuan, 030031, China*
† *E-mail: yanrong201712@163.com*

Abstract

In this talk, we establish and study a stochastic mosquito population suppression model incorporating the release of *Wolbachia*-infected males and time switching, where stochastic noises are given by independent standard Brownian motions. By combining the actual mosquito control strategy in Guangzhou, we assume that the waiting release period T between two consecutive releases of *Wolbachia*-infected males is less than the sexually active lifespan T of them. The existence and uniqueness of global positive solutions and stochastically ultimate boundedness for the stochastic model are obtained. Some sufficient conditions for the extinction and the existence of stochastic non-trivial periodic solutions are established. Furthermore, we assume that the release function is a general periodic function and some stochastic dynamical behaviors are obtained. Numerical examples are presented to illustrate the theoretical results.

References

[1] J. Yu, J. Li, Global asymptotic stability in an interactive wild and sterile mosquito model, J. Differ. Equ. 269 (2020) 6193-6215.

[2] J. Yu, Existence and stability of a unique and exact two periodic orbits for an interactive wild and sterile mosquito model, J. Differ. Equ. 269 (2020) 10395-10415.

[3] R. Yan, Q. Sun, Uniqueness and stability of periodic solutions for an interactive wild and *Wolbachia*-infected male mosquito model, J. Biol. Dyn. 16 (2022) 254-276.

[4] R. Yan, B. Zheng, J. Yu, Existence and stability of periodic solutions for a mosquito suppression model with incomplete cytoplasmic incompatibility, Discrete Contin. Dyn. Syst., Ser. B 28 (2023) 3172-3192.

Evaluating the intensity of a potential yellow fever outbreak during an international trading event: A case study on Canton Fair

Yining Chen*†1

[1] *Guangdong Polytechnic Normal University, China*
† *E-mail: chenyining87@163.com*

Abstract

We evaluate the risk of yellow fever outbreaks in a major trade event, with a case study of Canton Fair (Guangzhou, China), caused by case importation at different stages of the trade event. Our baseline model is a standard vector-borne disease transmission dynamics system, but we incorporate the division of a calendar year into favorable and unfavorable seasons based on impacts of different climatic conditions (temperature in particular) on mosquito population dynamics. We also incorporate square-waves to describe scenarios of case importation. We then use this periodic switching model to inform the potential of outbreaks and intensity of outbreaks due to case importation in different periods in relation to the two seasons. Our results show that importation of cases (even with a single case introduced) in the favorable season can induce a large outbreak in the local population in the host city, and the intensity of outbreak depends on the total number of imported cases (up to a level, when local transmission dominates). We also incorporate the public health interventions-isolation and emergency vaccination-to the model to provide quantitative information for the event organizer and public health decision makers for the preparedness and rapid response to the outbreak induced by case importation. This is a joint work with Yufeng Wang, Jianshe Yu and Jianhong Wu.

Special Session 26

Forced waves and spreading dynamics to the Lotka-Volterra competition system (a road- led environment) with climate changes

Hongyong Wang*†[1,2]

[1]*School of Mathematics and Statistics, Hunan First Normal University, Changsha 410205, China*
[2]*School of Mathematics and Physics, University of South China, Hengyang, 421001, China*
†*E-mail: sysuwhylj@aliyun.com*

Abstract

We studied the existence, uniqueness, and stability of the forced traveling waves for the Lotka-Volterra competition system with climate change. We show that the forced wave for the system exists and is unique, when the forced speed lies in a specific interval [3, 4]. Explicit expressions for the two end points of this interval were derived, and our finding indicates that they are related to the Fisher-KPP-type invasion speed. Furthermore, we establish a squeezing theorem to show the local stability of the forced waves. Finally, with the aid of the comparison principle, we establish the global stability of the forced waves when the initial data are properly assigned. On the other hand, motivated by the recent work of Berestycki and his research group [1, 2], we study the joint influence of a road-field diffusive system coupled with climate change on the dynamical behaviors of two competitive populations in the case where one species is assumed to be home territory. The forced traveling wave solution (TWS) is first investigated and we find that its existence depends on the value of the shift speed c. The forced TWS exists if and only if $c \in (0, c^*)$ (c^* is a threshold shifting speed and its value is related to the ratio of the two diffusion coefficients on the road and in the field). To draw a complete picture of the propagation dynamics, we investigate the global spreading behaviors in the case $c \in (c^*, \infty)$. In this case, the spatial domain resulting in the extinction of both populations is established, and this phenomenon can be interpreted as gap formation between the two biological species.

The spreading speed of a diffusive age-structured model in rapidly varying media

HAO KANG*†[1]

[1] *Center for Applied Mathematics, Tianjin University*
† *E-mail: haokang@tju.edu.cn*

Abstract

We establish the limit of spreading speed of a diffusive age-structured model with monostable nonlinearity in rapidly varying media. Moreover, by comparing the spreading speeds between this monostable reaction-diffusion equation with age structure and another one without age structure, we find that the former one is more complicated than the latter one and there even does not exist a determined relation between them.

The approach of bounded solutions of degenerate equations to traveling wave solutions

Wei-Jian Bo*†[1]

[1] *Xidian University*
† *E-mail: wjbo@xidian.edu.cn*

Abstract

In this talk, we consider the traveling waves and propagating terraces in a class of delayed diffusion equations with degenerate multistable nonlinearities. Due to the appearance of degeneracy and delay, the traditional methods such as the eigenvalue theory, phase plane analysis, spectral analysis and Evans function cannot be applied to prove the existence and stability of traveling waves. By introduction a novel method, we first obtain the alternative of traveling waves: either there is a monotone and unique, up to a translation, traveling wave connecting 0 and 1, or there is a unique propagating terrace connecting 0 to 1. Subsequently, we present a sufficient and necessary condition on the existence of traveling wave and propagating terrace, and establish the asymptotic behaviors of traveling wave and the sign of the unique wave speed. Very different as that in nondegenerate case, the traveling wave may decay exponentially or algebraically at the degenerate end and the unique wave speed may monotone decreasing or increasing in the delay under degenerate multistable case, which depend crucially on the sign of the unique wave speed without delay. Finally, we present the asymptotic stability of traveling wave, diverging traveling wave and propagating terrace, which depends on the degeneracy of nonlinearity and sign of the unique wave speed. We emphasize that the propagating terrace is asymptotical stable if the wave speeds in propagating terrace are not equal to each other and the intermediate connecting platforms are non-degenerate.

Inner spreading properties of several noncooperative integrodifference systems

Guo Lin[*†1]

[1] *Lanzhou University, China*
[†] *E-mail: ling@lzu.edu.cn*

Abstract

In this talk, I shall reconsider the spreading speed in several integro-difference systems. By applying the stability theory, this study establishes several sufficient conditions for successful inner spreading, which are notably less restrictive than those proposed in our previous research.

Propagation dynamics for lattice equation in a spatial-periodic shifting environment

Xuefeng San*†1, Zhi-Cheng Wang[2] and Ru-Xin Wu[2]

[1] *Chongqing University, China*
[2] *Lanzhou University, China*
† *E-mail: sanxf14@lzu.edu.cn*

Abstract

This talk deals with the propagation dynamics for lattice differential equations in a spatial-periodic shifting habitat. We first prove the existence, uniqueness and global exponential stability of the periodic forced KPP waves. Then we establish the existence of periodic forced pulse waves. We also investigate the spreading properties of the solutions. Our results indicate that the long-time behaviors of solutions depend on the speed of the shifting habitat and the minimum wave speed in the limiting environment.

Special Session 27

Hyperbolic geodesic flows vs Schrodinger operator: Subordinacy theory for long range operator

Disheng Xu*†1

[1] *Great Bay University*
† *E-mail: xudisheng@gbu.edu.cn*

Abstract

Inspired by the study of hyperbolic geodesic flows, we develop a comprehensive subordinacy theory framework for long-range operators on $\ell^2(Z)$, with several applications, bridging dynamical systems and spectral analysis. Joint work with Zhenfu Wang and Qi Zhou.

Extended states for multi-dimensional random Schrödinger operators with decaying Bernoulli potentials

Yunfeng Shi*†1

[1]*Sichuan University, China*
†*E-mail: yunfengshi@scu.edu.cn*

Abstract

In this talk, we will introduce recent progress on the delocalization for multidimensional random Schrödinger operators with decaying Bernoulli potentials.

Regularity of the IDS for quasi-periodic Schrodinger operators

Jing Wang[*†1]

[1]*School of Mathematics and Science, Nanjing University of Science & Technology, China*
[†]*E-mail: jing.wang@njust.edu.cn*

Abstract

In this talk, we will consider the regularity of the integrated density of states (IDS) for analytic quasiperiodic Schrodinger operators. On the one hand, in the subcritical regime, we proved that the IDS and Lyapunov exponent are log-Holder and Holder continuous for operators with dense Liouvillean frequencies, provided the analytic radius is large.On the other hand, we proved that the IDS is absolute continuous for operators with trigonometric potentials and Diophantine frequencies, provided the coupling constant is large, which are in the supercritical regime and partially answers an open problem posed by Eliasson in 2002. The main tools are quantitative almost reducibility and Aubry duality.This talk is based on joint works with Xu Xu, Jiangong You and Jiangong You.

Abstract continuity theorem for Lyapunov exponents of linear cocycles

Ao Cai*†1 and Xiaojuan Deng[1]

[1]*Soochow University, China*
†*E-mail: acai@suda.edu.cn*

Abstract

We prove Hölder continuity of Lyapunov exponents for general linear cocycles varying measures on the base in Wasserstein distance under the assumption of uniform large deviations type (LDT) estimates. This is a measure version of the abstract continuity theorem (ACT) established by Duarte-Klein. This continuity theorem is general and applicable to a wide range of mathematical models including product of random matrices and cocycles essentially generated by shifts. This is a joint work with Xiaojuan Deng (Soochow University).

Localization for quasi-periodic operators with C^2-cosine type and Lipschitz monotone potentials

Hongyi Cao*†[1], Yunfeng Shi[2] and Zhifei Zhang[1]

[1] *Peking University, China*
[2] *Sichuan University, China*
† *E-mail: 2001110004@stu.pku.edu.cn*

Abstract

In this talk, we present a method for proving Anderson localization using Rellich functions (parameterized eigenvalues) and Green's functions, which has applications in quasi-periodic operators on Z^d with C^2-cosine type and Lipschitz monotone potentials. This is based on joint works with Yunfeng Shi and Zhifei Zhang.

Special Session 28

A nonlocal reaction-diffusion pest model with impulsive IGRs treatments

Ruiwen Wu*†1

[1] *Jinan University*
† *E-mail: ruiwenwu@jnu.edu.cn*

Insect Growth Regulators (IGRs) are specific pesticide that impedes or disrupts normal development of pests, leading to reduced viability, death of individual insects, and ultimately extinction of the population. This study is to construct a nonlocal periodic delayed reaction-diffusion model to describe the effects of impulsive IGRs releasing on pests in a spatially and temporally heterogeneous environment. Furthermore, threshold dynamics of the model is built through dynamical system analysis. Besides, the numerical simulation of the model is carried out to verify the theoretical results, evaluate the influence of spatial heterogeneity and discuss the strategies of pest control. This study indicates that IGRs have significant effects in pest control, but the release strategies need to be carefully considered. Additionally, it is worth noting that if the influence of spatial heterogeneity is ignored in model construction, the risk of pest outbreaks may be misestimated.

Global dynamics of a degenerate reaction-diffusion model for Lyme disease with seasonal and multiple vector interactions

Fuxiang Li*†[1] and Ruiwen Wu[2]

[1] *China University of Geosciences, Beijing (CUGB), China*
[2] *Jinan University, China*
† *E-mail: fuxiang_li@cugb.edu.cn*

Abstract

In this talk, we construct a periodic degenerate reaction-diffusion model of Lyme disease with seasonal and spatial heterogeneous structure in both host and tick populations. Moreover, the tick population is divided into three stages–larva, nymph, and adult-each of which shows distinct feeding preferences for two vertebrate hosts. We introduce the basic reproduction ratio $R0$ for this model and establish that the disease-free periodic solution is globally asymptotically stable if $R0 < 1$, while the system admits a globally asymptotically stable positive periodic solution if $R0 > 1$. Numerically, we investigate the spread of Lyme disease in Bulgaria. Simulations support our analytical findings and highlight the important role that environmental heterogeneity plays in the transmission dynamics of Lyme disease.

Dynamical analysis of an almost periodic and age-structured population model

Jiawei Huo*†1

[1] *Beijing University of Chemical Technology*
† *E-mail: jiaweihuo@buct.edu.cn*

Abstract

The age-structured population model, as an important model in biomathematics, has been studied by many scholars in the past many years. It is well known that seasonal factors play an important role in population dynamics and are cyclic, largely predictable. However, variations in nature are hardly periodic and the almost periodicity is more likely to accurately describe natural fluctuations. Almost periodic functions, a generalization of periodic functions, are more suitable to characterize the population model. Therefore, we consider the classical age-structured population model in an almost periodic situation. The next generation operator is defined to decide the sign of the exponential growth bound of the model. Furthermore, we investigate the asymptotic behavior of the population model via strong ergodicity. This implies that the almost periodic age-structured population model has similar properties to the autonomous and periodic cases.

Special Session 29

Modeling the heterogeneous susceptibility and infectivity: A nonlocal state-structured SIR epidemic model

Lianwen Wang*†1

1 *Hubei Minzu University*
† *E-mail: lwwang@hbmzu.edu.cn*

Abstract

This report develops and analytically investigate a nonlocal state-structured SIR epidemic model governed by integro-differential equations, which characterizes heterogeneous host susceptibility and variable infectivity of infected individuals. Constructing appropriate Lyapunov functionals, we show that the basic reproduction number exclusively determines global stability of both disease-free and endemic steady states. Numerical simulations corroborate our theoretical findings and demonstrate the substantial impact of nonlocal kernels on disease transmission dynamics, besides, some significantly effective strategies are identified, including immunization of susceptible populations, contact-reduction measures, mitigation of pathogen-specific infectivity and retardation of disease progression. Furthermore, we apply an SVELITR model derived by discretizing the nonlocal state-structured model to project tuberculosis prevalence trends in China, providing quantitative insights into the efficacy of several interventions.

Stability analysis and optimal control for an age-structured tuberculosis model with pre- and post-exposure vaccinations

QIAN JIANG*†1

[1] *Sichuan University*
† *E-mail: jiangqian3987@163.com*

Abstract

In this work, we consider an age-structured TB model with pre- and post-exposure vaccinations, which is an extension of the models analyzed in [1,2] (Lietman et al. 2000, Bhunu et al. 2008). Especially, we present a systematic method for discussing the global stability, constructing a streamlined framework that facilitates both the optimal selection of kernel coefficients in Lyapunov functional and the rigorous verification of (semi-)negative definiteness for time derivatives. Additionally, we design a corresponding optimal problem to evaluate the impacts of pre- and post-exposure vaccinations on TB transmission dynamics. Analysis shows that, when the exposed population is too high, high-intensity post-exposure vaccination can be prioritized to rapidly reduce the number of infected populations and thus contain subsequent transmission.

The dynamics of predator-prey systems with cooperative hunting

YONG YAO*†1

1 *Wuhan Institute of Technology*
†*E-mail: mathyaoyong@163.com*

Abstract

In this talk, we consider the dynamics of predator-prey systems with different functional responses and cooperative hunting including the existence and qualitative properties of equilibria and various bifurcations such as the Bogdanov-Takens bifurcation and the degenerate Hopf bifurcation. The analysis results demonstrate that the consideration of cooperative hunting leads to relatively complex dynamics of predator-prey systems and appropriate intensity of cooperative hunting is beneficial for the persistence of predators and the diversity of ecosystem.

Stability of stochastic switching systems and its application to epidemic model

Jing Hu*†1

[1] *Ningxia University*
† *E-mail: hujing94@nxu.edu.cn*

Abstract

In this talk, our concern is to investigate the stochastic asymptotic stability of semi-Markovian hybrid switching systems. A hybrid process includingsemi-Markov process and mode-dependent fixed dwell time is introduced. By multiple Lyapunov function approach and ergodicity of the hybrid process, we obtain sufficient conditions to ensure the stability of the switching system. To explore the influence of state changes on brucellosis, a stochastic brucellosis model with semi-Markovian switchings and diffusion is proposed. Then we study the influence of stationary distribution of semi-Markov process on extinction of brucellosis in switching environment including both stable states, during which brucellosis dies out, and unstable states, during which brucellosis persists.

Special Session 30

The existence of homoclinic solutions for a class of second order Hamiltonian systems in one-dimensional

Fangfang Liao*†[1]

[1] *Xiangnan University, China*
† *E-mail: liaofangfang1981@126.com*

Abstract

In this talk, by employing sophisticated analysis, including establishin a version of the Gagliardo-Nirenberg inequality in one-dimensional and the Brezis-Lieb lemma, we prove that existence of homoclinic solutions for the second order Hamiltonian systems with an L^2-constraint under the three cases respectively.

Bifurcation structure of a symmetrically decoupled system

Laadjal Baya*†1

[1] *Laboratoire de mathématiques Université de Biskra, Algeria*
† *E-mail: baya.laadjal@univ-biskra.dz*

Abstract

In recent years, many works have been done on discrete dynamical systems represented by endomorphisms, which means that the given map is smooth, but does not have a uniquely defined inverse. However, it is worth noting that the description of evolutive phenomena in several applications leads to mathematical models represented by discrete dynamical noninvertible maps. It is particularly the case in adaptive control systems where one finds multistability, and noninvertibility plays an important role in the structure of the basins of attraction of the coexisting attractors, which may consist of disconnected regions. Generally, knowledge of the structure of the basins of attraction is key to understanding the long-term evolution of the system. Other applications that give rise to noninvertible maps include models from economics, radiophysics and neural networks. Throughout this paper, we use an example, namely the two-parameter family of planar non- invertible maps, to explain the basin behavior, its dynamics, stability and attractors.

Homoclinic solutions for a discrete periodic Hamiltonian system with perturbed terms

Yanshan Chen*†1

1 *Guangzhou University, China*
† *E-mail: yschen@e.gzhu.edu.cn*

Abstract

The purpose of this work is to obtain, in a more optimal way, sufficient conditions for the existence of at least one homoclinic solution to a periodic discrete Hamiltonian system with perturbed terms. In this work, p and L are T-periodic with respect to $n \in Z$, and they are not required to be positive definite. Whether at infinity or at the origin, the nonlinearity need not be superquadratic, but can be asymptotically quadratic or a mixture of them. This character of superquadraticity is essential as observed in previous literature. Moreover, the existence of homoclinic solutions is shown to remain when the discrete Hamiltonian system involves some perturbations $h \in l^1$. To the best of our knowledge, this is the first attempt to obtain the existence of homoclinic solutions for a perturbed discrete Hamiltonian system in the case where p and L are non-positive definite. Our work is an improvement and complement to previous work on the existence of homoclinic solutions for discrete or continuous Hamiltonian systems. Furthermore, our superior results may be applicable to other variational problems.

Dynamic adjustment mechanisms in cooperation for evolving strategy using differential equations and replicator dynamics

Chenxi Jiang*†1

[1] *Xiangnan University*
† *E-mail: jiangchenxi@xnu.edu.cn*

Abstract

This study investigates dynamic adjustment mechanisms in strategic cooperation among governments, enterprises, and executives, focusing on the dual goals of environmental conservation and economic growth. A mathematical model is developed using differential equations and replicator dynamics to capture the evolution of strategies and interactions between these entities. The findings indicate that when the government's regulatory benefits are less than the subsidy costs, enterprises resort to passive strategies, whereas stronger incentives for active participation lead to enhanced environmental actions. The results contribute to the mathematical understanding of cooperation dynamics and offer practical implications for improving ESG performance in real-world applications.

Breaking smoothness barriers: A time scale approach for fractional differential equation

Hua Kong*†1

[1] *College of Mathematics, Sichuan University, China*
† *E-mail: konghua2008@126.com*

Abstract

This talks suggests a time scale numerical method for fractional differential equations without initial value's smooth assumptions. First, the traditional numerical methods face challenges in initial value problems. Then, a fractional difference equation approach is employed to address the initial value non-smoothness's problem. The convergence result is provided. Finally, the asymptotic behavior of the nabla discrete Mittag-Leffler function is discussed, and the stable conditions for the numerical scheme of the time-fractional diffusion equation are explicitly given. Since there are no smooth assumptions or regularization conditions, the method can be extensively employed in initial value problems of fractional differential equations.

Special Session 31

Qualitative analysis and controllability of quaternion numerical time-delay systems

Jinrong Wang*†1

[1] *Guizhou University, China*
† *E-mail: jrwang@gzu.edu.cn*

Abstract

This report introduces the origin, application and characteristics of quaternions. For quaternion numerical time-delay difference systems, by introducing appropriate quaternion discrete matrix delay exponents, we derive the exact solution of linear time-delay difference systems and establish necessary and sufficient conditions for the controllability of linear systems.

We propose the concept of quaternion reachable division ring and utilize the left linear independence of quaternion vectors to overcome the difficulty caused by the non-commutativity of quaternion matrix multiplication (where $(AB)^T \neq B^T A^T$ in general). This allows us to establish a rank criterion for the controllability of linear systems.

For quaternion numerical time-delay differential systems, we introduce appropriate quaternion delay matrix exponential functions to derive exact solutions. We provide:

- Necessary and sufficient conditions for linear system controllability
- Sufficient conditions for nonlinear system controllability

The concept of maximal right linear independent group of quaternion vectors is proposed, overcoming the limitation that the Cayley-Hamilton theorem doesn't hold for quaternions, leading to a new rank criterion for linear system controllability.

Limit cycles of piecewise polynomial Hamiltonian systems

Tao Li*†1

[1] *Southwestern University of Finance and Economics, China*
† *E-mail: litao@swufe.edu.cn*

Abstract

We present advances in the study of limit cycles for piecewise polynomial Hamiltonian systems, addressing systems with both linear and nonlinear switching boundaries. Our work extends classical results to this more general class of discontinuous systems.

Pullback measure random attractors of lattice FitzHugh-Nagumo systems

Weisong Zhou*†1

[1] *Chongqing University of Posts and Telecommunications, China*
† *E-mail: zhouws@cqupt.edu.cn*

Abstract

We investigate the existence of random attractors for stochastic FitzHugh-Nagumo equations on infinite lattices with multiplicative white noise. Using appropriate transforms, we:

1. Demonstrate the existence of an absorbing set
2. Prove the random dynamical system is asymptotically compact
3. Establish the existence of the random attractor

Impulsive control to a periodic plant-pest-natural enemy model in a patchy environment

Fei Shi*†[1]

[1] *Chongqing Normal University, China*
† *E-mail: freestone33@163.com*

Abstract

For better economical balance, biological control (releasing natural enemies) and chemical control (spraying pesticide) are often conducted together to combat agricultural pests. These controls are usually applied at different times. In consideration of the above factors, we propose a time-periodic plant-pest-natural enemy model in a patchy environment with impulsive controls being applied at different fixed moments. Mathematical properties such as the 'pest-free' periodic solution, local stability and persistence are investigated. Moreover sufficient conditions ensuring the global asymptotically stability of the 'pest-free' periodic solution are derived. Finally, two illustrative examples and their numerical simulations are given to verify the effectiveness of our conclusion.

Special Session 32

Contact points and their bifurcations in planar singular perturbation problems

Jianhe Shen*†1

[1]Fujian Normal University, China
†E-mail: jhshen@fjnu.edu.cn

Abstract

This talk will introduce the notions on normal hyperbolicity of critical manifolds and contact points for singular perturbation problems. Then, based on geometric singular perturbation theory and the technique of blow-up, the classifications of flow near sereval typical contact points in planar singular perturbation problems are also introduced.

Resident-mutant dynamics: The role of averaging principle

Yuhua Cai*†1

[1]*Fujian Normal University, China*
†*E-mail: yuhua.cai@fjnu.edu.cn*

Abstract

In this talk, I briefly introduce the resident-mutant dynamics of similar strategies in fluctuating environments. Then, I focus on some new results for the resident system with multiple invariant probability measures. The global dynamics are fully captured through the averaging principle and the invasion criteria.

Canard explosion of a singularly perturbed SIS epidemic model with non-monotone incidence rate and Logistic-type population birth rate

Xiao Wu*†1

[1]*Donghua University, China*
†*E-mail: xiaowu2022@dhu.edu.cn*

Abstract

In this paper, we consider a singularly perturbed SIS epidemic model with non-monotone incidence rate and Logistic-type population birth rate. The known results of epidemic model with non-monotone incidence rate are the Hopf bifurcation, the Bogdanov-Takens bifurcation, backward bifurcation, and so on. Our results show that the disease-related death rate γ is an important parameter. As parameter γ varies, the system undergoes different kinds of bifurcations such as a supercritical singular Hopf bifurcation, a canard explosion bifurcation, a homoclinic bifurcation, and so on. To be more detail, the small limit cycle expands to a relaxation oscillation cycle by a sequence of headless canard cycles and headed canard cycles, which are hyperbolically stable. When $k = (1 + \varepsilon d)^{-1}$, the headed canard cycles and the relaxation oscillation cycle are bearded near the boundary equilibrium. This is a new dynamic phenomenon in the singularly perturbed epidemic model. Furthermore, some numerical simulations including bifurcations diagrams and phase portraits are presented to support the theoretical results. These dynamics effectively shed light on the underlying mechanisms of recurrent disease outbreaks

Existence and stability of slow-fast traveling pulses in a chemical system with quintic nonlinearity

ZHENGKANG LI[*†1]

[1]China University of Mining and Technology, China
[†]E-mail: lizk1994@163.com

Abstract

This paper focuses on the existence, asymptotic behaviors and stability of slow-fast traveling pulses in a catalytic or electrochemical oscillation system with quintic nonlinearity. Inspired by the geometric singular perturbation theory and generalized rotated vector field, we provide the existence and wave speed for the traveling pulses with slow and fast dynamics induced by quintic nonlinearity with a parameter a in such chemical system. More precisely, we can provide the parameter interval for the occurrence of figure-eight type double homoclinic cycles corresponding to the bright and dark slow-fast traveling pulse. Furthermore, the asymptotic behaviors of such slow-fast traveling pulses are exhibited by applying the asymptotic theory, and the nonlinear stability of slow-fast traveling pulses have been proved by the spectral theory.

Multi-scale problems and limit cycles in piecewise-smooth predator-prey systems

Zirui Zhu*†1

[1] *Henan Normal University, China*
† *E-mail: zhuzirui960511@163.com*

Abstract

We investigate the generation of limit cycles for two types of piecewise-smooth continuous systems under different multi-scale problems and demonstrate different mechanisms for generating limit cycles under different cases. First, we establish a standard slow-fast problem for a piecewise-smooth Leslie-Gower predator-prey model. By combining geometric singular perturbation theory and bifurcation theory, we mainly analyzed the generation of limit cycles caused by nonsmooth bifurcations. On the other hand, we extends to a Gauss-type predator-prey piecewise-smooth model, which consists of a regular subsystem and a singularly perturbed subsystem. At this point, there are two nested crossing limit cycles, which are actually determined by the trajectory structures on both sides of the discontinuous boundary. Subsequently, we numerically simulated the coexistence of more than two limit cycles, which is the result of the presence of the discontinuous boundary.

Special Session 33

Discontinuous Galerkin methods for PDEs with polynomial growth

Kele Sun*†[1]

[1] *Fudan University, China*
† *E-mail: 24110890009@m.fudan.edu.cn*

Abstract

Based on the framework of complex systems theory, we investigate high-order accurate numerical approximation methods for a class of semilinear parabolic partial differential equations (PDEs), addressing the PDE solving problems in control optimization and nonlinear systems. A numerical scheme using the local discontinuous Galerkin (LDG) method is established, with theoretical optimal error estimates of order $(k+1)$ in the L^2-norm provided, supported by several numerical examples validating the theoretical analysis. Furthermore, considering stochastic problems in complex systems, we extend the LDG method to stochastic partial differential equations (SPDEs), presenting numerical solution algorithms and time discretization schemes for the stochastic case. Numerical experiments demonstrate the $(k+1)$-order accuracy of the method under stochastic conditions. The research results provide theoretical guarantees and numerical implementation pathways for solving complex systems.

Neural transfer learning: A framework for creating nonlinear dynamical systems with prescribed behaviors

He Ma*†[1]

[1] *Fudan University, China*
† *E-mail: hma23@m.fudan.edu.cn*

Abstract

Data-driven techniques, particularly those leveraging machine learning, have proven effective in reconstructing nonlinear dynamical systems from data, primarily for tasks like forecasting and control. However, most existing efforts focus on modeling known systems rather than creating new ones. This talk presents Neural Transfer Learning (NTL), a neural network-based framework inspired by image style transfer. NTL fuses the 'content' and 'style' of multiple parent systems to generate entirely new dynamical systems with prescribed behaviors. The framework is validated on chaotic Lorenz-like systems, ecological resource-consumer-predator models, and epidemiological SEIRS models. The generated systems are characterized using Lyapunov exponents, power spectra, system-specific metrics, and attractor reconstructions. Results suggest that NTL offers a novel perspective for creating and analyzing nonlinear dynamical systems that are both structurally interpretable and dynamically diverse.

Global solutions to the inviscid Oldroyd-B equations

Zhaonan Luo*†1

[1]*Sun Yat-sen University, China*
†*E-mail: luozhn7@mail.sysu.edu.cn*

Abstract

In this talk we will present the global existence of solutions, optimal time decay rates, and optimal damping vanishing rates for the inviscid Oldroyd-B models with central diffusion. First, we will study the two-dimensional inviscid Oldroyd-B model and prove the global existence of a class of large data weak solutions in the co-rotational case, as well as small data weak solutions in the non-co-rotational case without damping. Additionally, we obtain the optimal decay rates for weak solutions in the non-co-rotational case. Next, we consider the decay of strong solutions for the two-dimensional non-co-rotational Oldroyd-B model without damping with a fractional central diffusion term. Finally, we investigate the problem of optimal time-uniform damping vanishing rates through global solutions with critical regularity for the non-co-rotational Oldroyd-B model that are uniform with respect to damping, along with their optimal decay rates.

Analytical and numerical results of PDE models from material sciences and biology

Xinyu Cheng*†1

1 *Fudan University, China*
† *E-mail: xycheng@fudan.edu.cn*

Abstract

Partial differential equations (PDE) often describe mathematical models of physical, biological, or financial phenomena. To study these PDEs, it is useful to consider approximate solutions via computer simulations; good accuracy and preserving intrinsic behaviors are crucial to design numerical schemes. In this talk we will give a brief introduction of two typical models, the phase field model and free boundary model, arising from the area of material sciences and biology respectively. Phase field models can be described as gradient flows therefore the energy dissipation (stability) provide a fidelity check for numerical schemes. We will introduce recent results in showing the energy stability for both traditional models and newly-developed trigonometric models. On the other hand, free boundary models have moving boundaries whose locations in time are to be solved. In this talk we will introduce a typical oxygen depletion model and give different analytic formulations to characterize such model armed with newly developed numerical schemes.

Bifurcations in shallow water equations with horizontal kinetic energy backscatter and bottom drags

JiChen Yang*†[1]

[1] *Harbin Engineering University, China*
† *E-mail: jichen.yang@hrbeu.edu.cn*

Abstract

Shallow water equations describe a thin layer of fluid bounded from above by a free surface, and from below by a rigid surface, near which the bottom drag plays a role in the dissipation of wave energy. Motivated by this and numerical schemes for large-scale geophysical flow, we consider the rotating shallow water on the whole space with smoothly and non-smoothly quadratic bottom drag terms, and in combination with horizontal kinetic energy backscatter terms built from negative viscosity and stabilising hyperviscosity with constant parameters. We prove the existence of nonlinear geostrophic equilibria and inertia gravity waves using Lyapunov-Schmidt reduction, and we study the bifurcations of nonlinear explicit flows that simultaneously solve the linear and nonlinear equations. We also discuss the impacts of bottom drags, energy backscatter, and Coriolis force on these flows. This is a joint work with Artur Prugger (Fraunhofer ITWM) and Jens Rademacher (Universität Hamburg).

Entropies of nonautonomous dynamical systems

Ying Wang*†1

[1] *Fudan University, China*
† *E-mail: yingwang19@fudan.edu.cn*

Abstract

In the first part, we study topological and measure-theoretical entropies of a nonautonomous dynamical system via the ideas of local entropy theory. We prove local and global variational inequalities, which relates to the topological entropy of a nonautonomous dynamical system to its measure-theoretical entropy. In the second part, we introduce weak expansiveness for a nonautonomous system following the ideas of Bowen and Misiurewicz. For further understanding of the weak expansiveness of a nonautonomous system, we also introduce and study the concept of topological conditional entropy for such class of dynamical system along the lines of Misiurewicz. We give a simple link relating weak expansiveness of a nonautonomous system to its topological conditional entropy. This is a joint project with Kexiang Yang, Guohua Zhang.

Special Session 34

Impact of degree correlation on SIR epidemic dynamics in complex networks

Yi Wang*†1

[1]*School of Mathematics and Physics, China University of Geosciences (Wuhan), China*
†*E-mail: yiwang@cug.edu.cn*

Abstract

Many real-world networks have nontrivial degree correlations. For example, many social networks show that high degree nodes tend to preferably connect with other high degree nodes, the so called "assortative mixing" property. In this topic, I will first compare SIR epidemic dynamics on configuration type networks with that on degree correlated networks; then formulate an edge-based SIR epidemic model on degree-correlated networks, and further discuss the relationship between the basic reproduction number on configuration type networks and that on degree correlated networks. In addition to present extensive numerical simulations, we provide some rigorous results. Finally, I briefly introduce our ongoing works on this topic.

Turing patterns in simplicial complexes

Shupeng Gao*[†1]

[1] *Tangshan Research Institute, Beijing Institute of Technology, China*
[†] *E-mail: spgao96@163.com*

Abstract

In this talk, we focus on the spontaneous emergence of organized patterns in systems with higher-order interactions. Natural patterns such as stripes and spots can be mathematically described by reaction-diffusion systems, a concept first introduced by Alan Turing in the early 1950s. These so-called Turing patterns have since become a cornerstone in understanding self-organization across physical, chemical, and biological systems. With the rise of network science, the study of pattern formation has expanded beyond traditional nearest-neighbor diffusion to include long-range and irregular connections. This shift has revealed new layers of complexity, including the emergence of spatiotemporally irregular patterns. In this context, we investigate Turing pattern formation on simplicial complexes—a class of higher-order structures where interactions can involve three or more nodes simultaneously, rather than just pairs. Higher-order interactions have recently gained prominence in modeling collective behavior in social and biological systems. Understanding how such interactions influence pattern formation is therefore a crucial challenge. We show that canonical reaction-diffusion systems defined on simplicial complexes give rise to fundamentally different pattern dynamics compared to traditional pairwise networks. For instance, we identify stable Turing patterns in which the fraction of nodes with reactant concentrations above equilibrium exhibits an exponential dependence on the average degree of 2-simplices. Moreover, we uncover parameter regimes where pattern formation occurs exclusively due to higher-order interactions—patterns that would not arise in systems with only pairwise links.

Optimal mixed control of networked reaction-diffusion systems

Xiaofeng Luo*†[1]

[1] *School of Mathematics, North University of China, China*
† *E-mail: luoxiaofeng@nuc.edu.cn*

Abstract

We introduce a control scheme able to steer the evolution of networked reaction-diffusion systems toward any intended dynamics. Specifically, we consider simplicial complexes of reaction-diffusion systems that can be collapsed into weighted complex networks by leveraging the linear relationship between the Laplacian matrix of the simplicial complex and the Laplacian matrix of each order simplex. The method integrates reaction and network-based diffusion controls and, as far as effectiveness and cost are concerned, features a significantly improved performance over traditional single-variable control methods. Numerical simulations using the FitzHugh-Nagumo and susceptible-infected-removed models demonstrate the resilience of the proposed mixed control across diverse systems, highlighting the potential of the method for complex system management and intervention strategies.

Research on complex spatiotemporal patterns in vegetation systems of arid and semiarid regions

Jing Li*†1

[1] *Shanxi University of Finance and Economics, China*
† *E-mail: Jingli_2016@126.com*

Abstract

In arid and semi-arid regions, vegetation often exhibits diverse spatial patterns, among which the typical structure is characterized by periodic Turing patterns (PP state). This study investigates the mechanisms underlying the formation of non-typical patterns based on a vegetation-water interaction model. By deriving the amplitude equation, we obtain the criteria for the occurrence of a subcritical Turing bifurcation, The analysis reveals a bistable phenomenon in which the bare soil state (BS state) and the uniform vegetation state can coexist stably. More importantly, under subcritical Turing bifurcation conditions, a snaking structure emerges in the bifurcation diagram, leading to the formation of localized patterns, spatial coexistence of uniform and periodic states within the same domain. The presence of these localized patterns enhances the ecological resilience of the vegetation system and helps prevent its degradation into desertified states. Moreover, the structure of localized patterns is highly sensitive to initial conditions. This study contributes to a deeper understanding of the mechanisms behind vegetation pattern formation in arid and semi-arid regions and provides theoretical support for desertification prevention and ecological management.

Mathematical analysis for stochastic model of Alzheimer's disease

Yongxin Zhang*†[1]

[1] *School of Mathematics, North University of China, China*
† *E-mail: zhangyongxin@nuc.edu.cn*

Abstract

Alzheimer's disease is a worldwide disease of dementia and is characterized by β-amyloid plaques. Increasing evidences show that there is a positive feedback loop between the levels of β-amyloid and Ca^{2+}. In this talk, stochastic noises are incorporated into a minimal model of Alzheimer's disease which focuses upon the evolution of β-amyloid and Ca^{2+}. Mathematical analysis indicates that solutions of the model without stochastic noises converge either to a unique equilibrium or to bistable equilibria. Analytical conditions for the stochastic P-bifurcation are derived by means of technique of slow-fast dynamical systems. A formula is presented to approximate the mean switching time from a normal state to a pathological state. A disease index is also proposed to predict the risk to transit from a normal state to a disease state. Further numerical simulations reveal how the parameters influence the evolutionary outcomes of β-amyloid and Ca^{2+}. These results give new insights on the strategies to slow the development of Alzheimer's disease.

Spike patterns in a vegetation model with cross-diffusion and saturated water absorption effect

Hongtao Zhang*†1

[1]*School of Mathematics, North University of China, China*
†*E-mail: zhanghongtao@nuc.edu.cn*

Abstract

Desertification is one of the most serious environmental problems in the world, and its expansion has not been fundamentally controlled in arid and semi-arid areas, where the spatial distribution of vegetation is crucial for maintaining ecological balance and is directly related to the stability of the ecosystem and the effectiveness of desertification control. The spike vegetation pattern optimizes the utilization of scarce water resources through the concentrated distribution of local high-density vegetation, reflecting the self-organizing adaptive mechanism of arid ecosystems. Its spatial structure evolution can serve as a key indicator for early warning of desertification. Based on a vegetation model with saturated water absorption effect, this study reveals the existence mechanism of the spike pattern. Specifically, nonlinear analysis shows that the spike pattern can be approximately formed by the superposition of multiple Turing modes. Its localized characteristics are verified by the same-order relationship between the spike radius and the square root of the diffusion coefficient and spectral analysis. The difference in spike characteristics under different diffusion forms reveals the regulatory mechanism of the diffusion mode on the pattern formation. Sensitivity analysis shows that the local diffusion system is more sensitive to environment disturbances than the degenerate system and the non-local diffusion system, which will provide a theoretical basis for the regulation of ecosystem stability. This study deepens the understanding of the formation mechanism of vegetation patterns in arid and semi-arid areas and provides scientific support for desertification risk warning and ecological restoration strategies.

Stochastic dynamics of an SEIR epidemic model on heterogeneous networks: A case of COVID-19

Xiaojie Jing*†1

[1] *School of Mathematics, North University of China, China*
† *E-mail: jingxiaojie@nuc.edu.cn*

Abstract

In this work, a stochastic SEIR epidemic model on heterogeneous networks is established, and the law of large numbers and the central limit theorem of the epidemic process are obtained. Using the random time transformation, the mean behavior of the epidemic process is analyzed, that is, the solution of the deterministic model is given. Further, the asymptotic distribution of the final size is provided. Then, the network-based stochastic epidemic model is applied to a COVID-19 infection at a construction site in Qingpu District of Shanghai, and the parameters of the model are estimated by fitting the data of confirmed cases. Based on the estimated parameter values, the intervention measure implemented at the site is assessed by numerical simulations, and we find that the intervention does not effectively curb the development of the disease. In addition, simulation results show that the asymptotic approximation for the final size is good. The impact of the detecting or symptomatic rate on the final size is also analyzed by numerical studies. The results indicate that as the rate increases, the mean of the final size decreases and the variance increases, which is more conducive to controlling the spread of the disease.

Special Session 35

Uniform measure attractors for non-autonomous stochastic delayed lattice systems with high-order nonlinear noise

Dingshi Li*†1

1 Southwest Jiaotong University, China
† E-mail: lidingshi2006@163.com

Abstract

This paper studies uniform measure attractors for nonautonomous stochastic lattice systems with delay driven by high-order nonlinear noise. While previous studies have investigated attractors for stochastic lattice systems with delay, the existence of uniform measure attractors for systems with high-order nonlinear drift and diffusion terms remains unresolved due to the inherent difficulty in obtaining uniform closed absorbing sets under high-order nonlinearities. To address this challenge, we establish an equivalent theoretical framework for uniform measure attractors via ω-limit compactness and uniform asymptotic tightness, which removes the reliance on uniform closed absorbing sets. Within this novel framework, we prove the existence and uniqueness of uniform measure attractors for non-autonomous stochastic delay lattice systems with almost-periodic forcing and high-order nonlinear terms.

Variable-step discretizations of Volterra equations with completely positive kernels

Lei Li*†1

[1] *Shanghai Jiao Tong University, China*
† *E-mail: leili2010@sjtu.edu.cn*

Abstract

We investigate the discretization of Volterra integral equations on nonuniform meshes. We first consider equations with completely positive kernels and give descriptions to such kernels for nonuniform meshes using the resolvent kernels. Second, we investigate the monotonicity properties of the solutions for equations in R with nonincreasing and completely positive kernels. Using the resolvent kernels, we show the monotonicity preserving properties of the numerical solutions on nonuniform meshes.

Discretization of super-linear slow-fast stochastic differential equations

Xiaoyue Li*†1

[1] *Tianjin Polytechnic University, China*
† *E-mail: lixy@tiangong.edu.cn*

Abstract

This talk is dedicated to the discretization of super-linear slow-fast stochastic differential equations (SFSDEs). Borrowing the heterogeneous multiscale idea, we propose an explicit multiscale discrete equation suitable for SFSDEs with locally Lipschitz coefficients using an appropriate truncation technique. By the averaging principle, we establish the strong convergence of the numerical solutions to the exact solutions in the pth moment. Additionally, under lenient conditions on the coefficients, we also furnish a strong error estimate. This work is accomplished in cooperation with Yuanping Cui and Xuerong Mao.

Viscosity solution approach to continuous-time decision processes with history-dependent policies

Zhongwei Liao*†1

[1] *Beijing Normal University, China*
† *E-mail: zhwliao@bnu.edu.cn*

Abstract

This paper investigates the optimal control problems for the finite horizon continuous time decision processes with history-dependent control policies. We develop the compactification method to show the existence of optimal policies based on the application of Skorokhod's representation of Markov chains. Then, the value function is characterized as the unique viscosity solution to certain differential-difference Hamilton-Jacobi-Bellman equation. This work is accomplished in cooperation with Yanhua Mi and Jinghai Shao.

Truncated Euler-Maruyama method and its positivity of stochastic L-V competition models

Fengying Wei*†1

1 *Fuzhou University, China*
† *E-mail: weifengying@fzu.edu.cn*

Abstract

In this talk, we aim at the well-known stochastic Lotka-Volterra model with the interaction of multi-species in ecology of having some typical features: highly nonlinear, positive solution and multi-dimensional. The known numerical methods including the tamed/truncated Euler-Maruyama (EM) applied to it do not preserve the positivity of the stochastic L-V competition models. The aim of this talk is to modify the truncated EM to establish a new positive preserving truncated EM (PPTEM). To simplify the proof as well as to make our theory more understandable, we first develop a nonnegative preserving truncated EM (NPTEM) and then establish the PPTEM. Of course, we should point out that the NPTEM has its own right as many SDE models in applications have their nonnegative solutions. This is a joint work with Xuerong Mao and Teerapot Wiriyakraikul.

Special Session 36

Synchronization of neural networks under DoS attacks via multi-channel discrete control

Shuqing Gong*†1

1 *School of Mathematics and Statistics, Changsha University of Science and Technology, China*
† *E-mail: sqgong2019@163.com*

Abstract

This work focuses on the synchronization problem of neural networks with time delay under denial-of-service (DoS) attacks through a multi-channel discrete control approach. The control information updates in the discrete control are aperiodic, but there exists a uniform upper bound on the time interval between two consecutive updates. The DoS attack active interval is uncertain and can independently disrupt each transmission channel. These attacks can force the discrete control to discard outdated information, which leads to control task failures. Under such circumstances, a strategy is proposed to transform the discrete control into switching control. By using switched system theory and constructing a piecewise Lyapunov function, sufficient conditions for the duration and frequency of DoS attacks are derived. Furthermore, the criteria with LMIs form on the network parameter are established to guarantee synchronization of the neural networks. Finally, the effectiveness of the proposed theory is demonstrated through an example and its numerical simulations.

Planar Schrödinger equations with critical exponential growth

Lixi Wen*†1

[1] *Changsha University of Science and Technology, China*
† *E-mail: wlxcsust@csust.edu.cn*

Abstract

In this talk, we will introduce some recent results for elliptic equations with exponential critical growth. By using variational methods and subtle estimates, we establish the existence, multiplicity and concentration of solutions for different elliptic equations.

Geometric theory of distribution shapes for autoregulatory gene circuits

Ying Sheng*†1

[1] *Guangzhou University, China*
† *E-mail: shengy7721@163.com*

Abstract

In this study, we provide a complete mathematical characterization of the phase diagram of distribution shapes in an extension of the two-state telegraph model of stochastic gene expression in the presence of positive or negative autoregulation. Using the techniques of second-order difference equations and nonlinear discrete dynamical systems, we prove that the feedback loop can only produce three shapes of steady-state protein distributions (decaying, bell-shaped, and bimodal), corresponding to three distinct parameter regions in the phase diagram. The boundaries of the three regions are characterized by two continuous curves, which can be constructed geometrically by the contour lines of a series of ratio operators. Based on the geometric structure of the phase diagram, we then provide some simple and verifiable sufficient and/or necessary conditions for the existence of the bimodal parameter region, as well as the conditions for the steady-state distribution to be decaying, bell-shaped, or bimodal. Finally, we also investigate how the phase diagram is affected by the strength of positive or negative feedback.

Inference of stochastic gene transcription model based on qualitative of data

LIANG CHEN*†1

[1] *Guangzhou University, China*
† *E-mail: chenliang.cq@foxmail.com*

Abstract

Current a large of research is based on fitting stochastic transcription data to estimate the frameworks and parameters of gene regulatory networks. However, traditional quantitative methods often deviate from the qualitative characteristics of the data and are greatly affected by the complexity of the model and the sample size. This report explores qualitative characteristics based on transcription data, develops qualitative analysis methods for related dynamical systems, conducts model screening, parameter estimation, and transcriptional regulation analysis, and reveals interactive regulatory mechanisms of signaling pathways, elucidating transcriptional data of bacteria, yeast, and mammalian cells.

Special Session 37

Discontinuous dynamics for impulsive differential systems with the state-dependent impulses

Xilin Fu*†1 and Liqin Zhang[1]

[1] *School of Mathematics and Statistics, Shandong Normal University, China*
† *E-mail: xilinfu@hotmail.com*

Abstract

Up to now, most of the research on impulsive differential systems focuses on the case with fixed time pulse, and rarely considers the case with arbitrary time pulse. The basic characteristic of the impulsive differential system with arbitrary time pulse is that its pulse depends on the state, which leads to "pulsation phenomenon" in the pulse surface. In this talk, from the viewpoint of discontinuous dynamical system, the pulse system which depends on state pulse is regarded as a discontinuous system bounded by a surface represented by pulse surface function. We use the measurement method of the discontinuous boundary flow transformation of discontinuous dynamical systems, build the mapping structure of the flow collision pulse surface by means of the characteristics of pulse mapping and pulse surface function, and the corresponding complex dynamical results of these systems are obtained.

Hopf bifurcation analysis for a maglev system with two time delays

Lingling Zhang*†1

[1] *Changsha University, China*
† *E-mail: z002005@163.com*

Abstract

We undertake the Hopf bifurcation analysis of a model for a rigid guideway maglev system with time-delayed feedback. First, the nonlinear dynamic model is presented with consideration of two delayed state feedback states. Then using the method combining the algebra and geometry, we explore the stability crossing curves in the two-delay parameter plane on which the characteristic equation has purely imaginary solutions. The multiple scales method is used to judge the stability of the limit cycle which may be caused by Hopf bifurcation. Exhaustive numerical simulation and experimental result demonstrate the complex behaviors of the guideway maglev system near the domain of stability.

Finite/fixed-time stability of discontinuous systems via Lyapunov method with indefinite derivative

ZENGYUN WANG*†1

[1] *Hunan First Normal University, China*
† *E-mail: zengyunwang@126.com*

Abstract

This talk focuses on the finite/fixed-time stability of nonautonomous differential equations with discontinuity. First, within the framework of the Filippov state solution, the finite-time stability (FNTS) and fixed-time stability (FXTS) problems of differential inclusions (DI) are investigated using the generalized Lyapunov functional method. These theoretical results are then applied to stabilize fuzzy switching neural networks. Second, under the framework of impulsive differential inclusions, the finite-time stability (FNTS) and fixed-time stability (FXTS) problems of discontinuous impulsive differential systems are studied using the generalized Lyapunov functional method with indefinite derivative. Additionally, several examples are provided to demonstrate the effectiveness of the proposed results. This work is joint with Zuowei Cai, Lihong Huang and Jinde Cao.

Dynamics analysis of two diffusive epidemic models with spatial heterogeneity

Lian Duan*†1

[1] *Anhui University of Science and Technology, China*
† *E-mail: lduan@aust.edu.cn*

Abstract

In this talk, I will report the threshold dynamics of two class of diffusive epidemic models with heterogeneity parameters. One is diffusive cholera epidemic model, and another is diffusive SIRI epidemic model. Our analyses show that the basic reproduction number which serves as a threshold parameter that predicts whether epidemics will persist or become globally extinct. For the latter, the asymptotic profiles of the positive steady state are also discussed.

Limit cycles in the perturbation of a piecewise cubic quasi-homogeneous system with a global center

Ting Chen*[†1]

[1]*Guangdong University of Finance and Economics, China*
[†]*E-mail: chenting0715@126.com*

Abstract

In this talk we study the quasi-homogeneous center problem and the bifurcation of limit cycles in piecewise smooth differential systems. Using the higher-order Poincare Lyapunov method it presents the explicit conditions for piecewise quasi-homogeneous cubic polynomial systems that exhibit a global center. Base on one of the center conditions, it proves that such piecewise system has at least 5 limit cycles bifurcating from the quasi-homogeneous global center. Moreover, by taking advantage of the inherent structure of this piecewise quasi-homogeneous cubic system, our result provides a general expression of its first-order Melnikov function under polynomial perturbations of degree n. Then it is utilized to determine the Poincare cyclicity in the piecewise smooth case. These findings serve to update and generalize existing results.

Special Session 38

Examine the efficiency of impulsive interventions on a network model

Yanyu Xiao*†[1]

[1] *University of Cincinnati, America*
† *E-mail: yanyu.xiao@outlook.com*

Abstract

We examined the transmission dynamics of certain preventable infectious diseases using a network-based model with impulsive control strategies. We derived a threshold value, established the global stability of the disease-free periodic solution, and demonstrated the persistence of the disease.

Dynamics of a epidemic system with diffusion and media impact

Wenjie Li*†[1]

[1] *Kunming University of Science and Technology, China*
† *E-mail: forliwenjie2008@163.com*

Abstract

This report studies an impact of media epidemic system with diffusion and linear source. We first derive the uniform bounds of solutions to impact on media reaction diffusion system. Then, the basic reproduction number is calculated and the threshold dynamics of impact media reaction diffusion system is also given and the Kuratowski measure κ of noncompactness is also considered. In addition, assume the spatial environment is homogeneous, it is shown that the unique endemic equilibrium of the system is global stability by constructing suitable Lyapunov function. Finally, we discuss the asymptotic profile of the system when the diffusion rate of the susceptible (infected) individuals for the system tends to zero or infinity. The main results show that the activities of infected individuals can only be at low risk, and then the virus eventually will be extinct, that is, to control the entry of viruses from abroad and increase the detection of domestic viruses. Finally, some numerical simulations are worked out to confirm the results obtained in this report.

Fixed-time multistability in switched fuzzy neural networks with multi-controller strategies

Shiqin Ou*†1

[1] *Guizhou University, China*
† *E-mail: sqou@hnu.edu.cn*

Abstract

This paper provides theoretical analysis of the fixed-time multistability in switched fuzzy neural networks, employing multi-controller strategies and a state-dependent switching mechanism. Utilizing the properties of M-matrix, Lyapunov functions method, and some inequality techniques, we establish some sufficient conditions to ascertain that the number of exponentially stable almost-periodic solutions can be up to 4^n, where n is the number of neurons. Furthermore, we design various controllers to achieve the fixed-time stability for various equilibria/almost-periodic solutions located in the positive invariant sets. Then, the settling time for the switched fuzzy networks to achieve multistability is estimated. The numerical examples are presented to demonstrate the theoretical results.

Dynamics of a discrete-time reaction-diffusion SIS epidemic model in heterogeneous environment

Ruiqiang Zhuo*†[1]

[1] *Guangzhou University, China*
† *E-mail: zhuoruiqiang@gzdx.wecom.work*

Abstract

Many infectious diseases show distinct spatiotemporal decoupling: evolutionary processes precede dispersal, and epidemiological data are usually collected at fixed time intervals, so discrete-time diffusion epidemic models may offer a more realistic representation of epidemiological dynamics compared to their continuous-time counterparts. This paper introduces a modeling framework for discrete-time reaction-diffusion epidemic systems in heterogeneous environments, with monotonicity property embedded in the proposed model. The variational characterization of the principal eigenvalue of an associated eigenvalue problem is established, based on which, we derive the variational characterization of the basic reproduction number R_0 and discuss its monotonicity with respect to the diffusion rate of infected individuals. Then, we give the existence, uniqueness and stability of steady states in terms of R_0. Finally, the asymptotic behaviors of the unique endemic equilibrium are analyzed when the diffusion rate of either the susceptible or infected individuals tends to infinity or zero. Our results reveal that the optimal strategy of eliminating the disease at least at low-risk sites is to restrict the diffusion of infected individuals. In particular, it is shown that the spatial heterogeneity can enhance disease transmission.

Special Session 39

Heteroclinic cycles and chaos in a class of 3D three-zone piecewise affine system

Qigui Yang*†1

[1]*South China University of Technology, China*
†*E-mail: qgyang@scut.edu.cn*

Little seems to be known about heteroclinic cycles and chaos in three-dimensional piecewise smooth dynamical systems with two or more discontinuous boundaries. This article presents a new class of three-dimensional three-zone piecewise affine systems with two discontinuous boundaries and provides some criteria for the existence of heteroclinic cycles in the following cases: (i) one saddle point and two focus points, (ii) two saddle points and one focus point, and (iii) three saddle points. Moreover, sufficient conditions for the existence of chaos are established. Finally, two numerical examples are provided to show the feasibility of our theoretical approach.

Bifurcations at infinity in general 3D piecewise-smooth quadratic vector fields

Yongjian Liu*†1

[1] *Yulin Normal University, China*
† *E-mail: liuyongjianmaths@126.com*

Abstract

Research on bifurcations at infinity in three-dimensional piecewise-smooth nonlinear vector fields remains scarce in the existing literature. This paper aims to bridge this gap by investigating bifurcations at infinity in general 3D piecewise-smooth quadratic vector fields. We derive the expression for the sliding vector field in the switching region at infinity utilizing the Poincare compactification and the Filippov convention. Then, we establish the conditions under which the 36-parameter family exhibits local codimension-0 singularities and codimension-1 bifurcations at infinity. These findings offer valuable insights into the complex dynamics of piecewise smooth nonlinear vector fields. Last but not least, the main results are applied to two models of three-dimensional variable-boostable chaotic flows, whose vector fields contain only square (e.g., x^2, y^2 and z^2) or cross terms (e.g., xy, yz and zx), and the phase portraits of the dynamic behaviors at infinity are described.

Bifurcations of quasiperiodic orbits and the emergence of strange nonchaotic attractors in a piecewise smooth map

Zhouchao Wei[*†1]

[1] *China University of Geosciences, China*
[†] *E-mail: weizhouchao@163.com*

Abstract

This paper investigates the formation and evolution of strange nonchaotic attractors (SNAs) in a noninvertible piecewise smooth map subjected to quasiperiodic forcing. The model is constructed as a two-dimensional skew-product system, combining a one-dimensional piecewise smooth base map with an irrational rotation. The presence of both a smooth critical curve and a discontinuity line generates multiple critical structures that shape the forward dynamics. By exploring representative parameter regimes, four distinct scenarios are identified: (i) interactions between smooth critical curves and absorbing regions; (ii) boundary changes due to the discontinuity; (iii) the combined influence of smooth and discontinuous critical sets; and (iv) coexistence and bifurcation cascades of high-period tori. For each scenario, numerical and analytical results demonstrate how the interplay between critical curves and preimage regions governs transitions from smooth tori to SNAs, leading to foldings, band splittings, and the formation of holes in the basin. Overall, the study shows that the combined effect of smooth folds and discontinuities determines the structure, evolution, and boundaries of attractors in this class of quasiperiodically forced noninvertible maps, extending previous results for purely smooth cases.

Measurably dominated splitting of fields of Banach spaces: Beyond the multiplicative ergodic theorem

Caibing Zeng*†1

[1] *South China University of Technology, China*
† *E-mail: macbzeng@scut.edu.cn*

Abstract

We establish a quasi-equivalent relationship between measurably contracting cone families and measurably dominated splittings in measurable fields of Banach spaces. Under an integrability condition, we derive a generalized Krein-Rutmann-type theorem for compact, injective linear cocycles on Banach spaces, without assuming cocycle compactness or integrability. The Lian-Wang index, rather than the Lyapunov norm, is employed to quantify contracting cone families and their eventual measurability. Leveraging smooth ergodic theory, we prove the existence of measurably dominated splitting in probability. Using the graph transform method, we further show that fields of Banach spaces admit measurably dominated splitting when cone invariance is satisfied.

Dynamics of fractional stochastic diffusive SIRS epidemic model with Levy noise

Zaitang Huang*[†1]

[1] *Nanning Normal University, China*
[†] *E-mail: zaitanghuang@163.com*

Abstract

Owing to the fractional diffusion described by a spectral fractional Neumann Laplacian, the nonlocal diffusion model can be used to address the spatiotemporal dynamics driven by the nonlocal dispersal. In this paper, we mainly concerned with spatiotemporal dynamics in fractional stochastic diffusive SIRS epidemic model with Levy noise. We first state the well-posedness of the problem via iterative approximations and energy estimates. Then, the existence and uniqueness of random attractors and invariant measures for the equations are established. Finally, a large deviation principle result for solutions of fractional stochastic diffusive SIRS epidemic model with Levy noise is obtained by the method of weak convergence. Interestingly, this shows the effect of fractional Laplacians which can stabilize or destabilize the system which is significantly different from the classical Laplace operators.

Regular

An n-dimensional strategic form game with dyadic interactions

Abdelrahim Mousa*†1

[1]*Birzeit University(BZU), Birzeit, Palestinian Territories*
†*E-mail: asaid@birzeit.edu*

Abstract

We present an n-dimensional strategic form game for finite types of homogeneous players, where dyadic interaction is the only allowed form of interaction. In this game-theoretic model, we determine two fulcrum vectors based on well-defined n-dimensional coherent strategic thresholds. We use the fulcrum vectors to determine all pure and probabilistic strategic vectors that form Nash and Bayesian Nash equilibria, respectively. Moreover, we identify the corresponding pure and probabilistic Nash regions in $\mathbb{R}^n$.

Keywords: Game theory, n dimensional normal form game, Pure and probabilistic strategies, Nash and Bayesian Nash equilibria

Managing the optimal strategies for an economic agent with uncertain lifetime

Salam Shrateh[1] and Abdelrahim Mousa[*†1]

[1] *Birzeit University (BZU), Ramallah, West Bank, Palestinian Territories*
[†] *E-mail: asaid@birzeit.edu*

Abstract

We introduce an extension continuous time model for the famous Merton's model by considering the problem of an economic agent whose lifetime is uncertain and aims to determine the optimal strategies concerning social welfare purchase, life insurance purchase, consumption and investment. The economic agent aims to maximize an expected utility obtained from family consumption, size of the estate in case of premature death, and size of the estate at retirement time. We assume that the economic agent has continuous access to a finite number of social welfare providers and life insurance providers. Meanwhile, we assume that the economic agent invests all her saving in a financial market consists of one risk-free asset and a finite number of risky assets. We restate the problem under consideration as one of the stochastic optimal control problems and apply the dynamic programming principle to derive a second-order nonlinear partial differential equation whose solution is the objective function for the problem under consideration. We manage to find an explicit solution for the underlying optimal strategies. Finally, we provide a numerical solution to show some properties for the optimal controls.

Keywords: Optimal consumption, Investment, Insurance and welfare strategies, Stochastic optimal control, Dynamic programming principle

Asymptotic stability of solutions to nonlinear difference equations with time-varying delay and periodic coefficients in linear terms

Inessa Matveeva*†1

1 *Novosibirsk State University, Novosibirsk, Russia*
† *E-mail: i.matveeva@g.nsu.ru*

Abstract

We consider systems of nonlinear difference equations with time-varying delay

$$x_{n+1} = A(n)x_n + B(n)x_{n-\tau(n)} + F(n, x_n, x_{n-1}, \dots, x_{n-\tau}), \qquad n = 0, 1, \dots, \qquad (1)$$

where $A(n)$, $B(n)$ are $m \times m$-matrices with N-periodic entries, $\tau(n)$ defines time-varying delay, $\tau(n) \in \mathbb{N}$, $1 \leq \tau(n) \leq \tau < \infty$, $F(n, u_0, u_1, \dots, u_\tau)$ is a continuous vector-function satisfying the estimate

$$\|F(n, u_0, u_1, \dots, u_\tau)\| \leq \sum_{j=0}^{\tau} q_j \|u_j\|^{1+\omega_j}, \qquad q_j \geq 0, \quad \omega_j \geq 0.$$

Using a special Lyapunov–Krasovskii functional, the asymptotic stability of solutions to linear systems of the form (1) ($F(n, u) \equiv 0$) was studied in [1]. This functional is a difference analog of the functional introduced in [2] for time-delay periodic differential equations. In the present paper, conditions for the asymptotic stability of the zero solution to (1) are established. We obtain estimates for attraction sets and estimates characterizing stabilization rates of solutions to (1) at infinity. This paper continues our investigations of nonautonomous differential-difference equations (for example, see [3–6]).

The work is supported by the Mathematical Center in Akademgorodok under agreement No. 075-15-2025-349 with the Ministry of Science and Higher Education of the Russian Federation.

Keywords: Nonlinear difference equations, Delay, Periodic coefficients, Asymptotic stability, Estimates for solutions

References

[1] Demidenko G. V., Baldanov D. Sh. Exponential Stability of Solutions to Delay Difference Equations with Periodic Coefficients // In: Continuum Mechanics, Applied Mathematics and Scientific Computing: Godunov's Legacy – A Liber Amicorum to Professor Godunov (Eds.: Demidenko G.V., Romenski E., Toro E., Dumbser M.). Cham, Switzerland: *Springer Nature*, 2020, 93–100.

[2] Demidenko G. V., Matveeva I. I. Stability of Solutions to Delay Differential Equations with Periodic Coefficients of Linear Terms. *Siberian Mathematical Journal*, **48:5** (2007), 824–836.

[3] Matveeva I. I. Estimates of the Exponential Decay of Solutions to Linear Systems of Neutral Type with Periodic Coefficients. *Journal of Applied and Industrial Mathematics*, **13:3** (2019), 511–518.

[4] Matveeva I. I. Estimates for Solutions to a Class of Nonautonomous Systems of Neutral Type with Unbounded Delay. *Siberian Mathematical Journal*, **62:3** (2021), 468–481.

[5] Matveeva I. I., Khmil A. V. Stability of Solutions to One Class of Difference Equations with Time-varying Delay and Periodic Coefficients in Linear Terms. *Mathematical notes of NEFU*, **30:4** (2023), 37–48.

[6] Matveeva I. I. Estimates of Solutions for a Class of Nonautonomous Systems of Neutral Type with Concentrated and Distributed Delays. *Computational Mathematics and Mathematical Physics*, **64:8** (2024), 1796–1808.

Bifurcation and chaos control in a nonlinear Cournot duopoly model with bounded rationality

Mourad Azioune*†1, Mohammed Salah Abdelouahab‡1 and René Lozi◇2

[1]*Laboratory of Mathematics and their interactions, Abdelhafid Boussouf University Center, Mila, Algeria*
[2]*Laboratory J.A. Dieudonné, CNRS, Université Côte d'Azur, France*
†*E-mail: m.azioune@centre-univ-mila.dz*
‡*E-mail: m.abdelouahab@centre-univ-mila.dz*
◇*E-mail: Rene.Lozi@univ-cotedazur.fr*

Abstract

In this study, we investigate the dynamics of a nonlinear discrete-time Cournot duopoly game with firms exhibiting bounded rationality. A system of two difference equations is employed to model their strategic interactions. Our analysis focuses on the existence and stability of equilibrium points. We observe that as the adjustment speed parameters increase, the Nash equilibrium undergoes a period-doubling bifurcation, resulting in more complex and chaotic market behaviors. This phenomenon is further explored through numerical simulations, where parameter variations reveal their impact on system dynamics. To address instability, we implement a state feedback control approach, introducing a controlling factor that effectively restores stability to the Nash equilibrium, as confirmed by simulations.

Keywords: Cournot duopoly, Discrete-time system, Bifurcation, Chaos control

References

[1] Amira, R., Abdelouahab, M. S., Touafek, N., Mesmouli, M. B., Zaidi, H. N., & Hassan, T. S. Nonlinear Dynamics in a Remanufacturing Duopoly Game: Spectral Entropy Analysis and Chaos Control. *AIMS Mathematics*, **9(3)**, 7711–7727.

[2] Li, B., Zhang, Y., Li, X., Eskandari, Z., & He, Q. Bifurcation Analysis and Complex Dynamics of a Kopel Triopoly Model. *Journal of Computational and Applied Mathematics*, **426**, 115089.

Criterions of dichotomy for differential-difference equations and applications

Gennadii Demidenko*†1

[1] *Novosibirsk State University, Novosibirsk, Russia*
† *E-mail: g.demidenko@g.nsu.ru*

Abstract

We consider systems of nonlinear differential equations

$$\frac{dy}{dt} = B(t)y + F(t,y), \quad -\infty < t < \infty, \tag{1}$$

and systems of difference equations

$$x_{n+1} = A(n)x_n + f(n, x_n), \quad n \in Z, \tag{2}$$

where $B(t)$ is an $(m \times m)$-matrix with T-periodic entries and the matrix sequence $\{A(n)\}$ consists of $(m \times m)$-matrices with N-periodic entries.

We formulate new criterions for the exponential dichotomy of (1), (2), obtain estimates for dichotomy parameters. Conditions for the existence of periodic solutions to these nonlinear systems are established and their stability is proved for small perturbations.

The work is supported by the Mathematical Center in Akademgorodok under agreement No. 075-15-2025-349 with the Ministry of Science and Higher Education of the Russian Federation.

Keywords: Periodic solutions, Exponential dichotomy, Lyapunov equations

References

[1] Demidenko G. V. On Conditions for Exponential Dichotomy of Systems of Linear Differential Equations with Periodic Coefficients. *International Journal of Dynamical Systems and Differential Equations*, **6:1** (2016), 63–74.

[2] Demidenko G. V. On One Class of Systems of Linear Differential Equations with Periodic Coefficients in Linear Terms. *Siberian Mathematical Journal*, **62:5** (2021), 805–821.

[3] Demidenko G. V., Dulepova A. V. On Periodic Solutions of One Second-order Differential Equation. *Journal of Mathematical Sciences*, **278:3** (2024), 314–327.

Designing a health-and-life insurance based on an SIH-type epidemic model involving two groups of individuals

Jonathan Hoseana*†1, Felivia Kusnadi[1], Gracia Stephanie[1], Levana Loanardo[1] and Catherine Wijaya[1]

[1] *Parahyangan Catholic University, Parahyangan Catholic University, institution, id, Bandung Indonesia, Indonesia*
† *E-mail: j.hoseana@unpar.ac.id*

Abstract

In a recent study, we designed a health-and-life insurance scheme based on a continuous SIH-type epidemic model, deriving expressions for the key financial quantities—the insurance's net and gross premiums, the insurer's minimum loss-preventing start-up capital, and the insurer's total profit—and discretizing our entire construct to facilitate numerical simulations. In the present study, we extend our design to the case where the epidemic model is developed into a multi-group form, accommodating two different groups of individuals possibly enrolling to two different insurance schemes. Our ongoing study progresses towards a generalization to an arbitrary number of groups of individuals.

Keywords: Health and life insurance, Epidemic model, Premium, Capital, Profit, Multi group

References

[1] J. Hoseana, F. Kusnadi, G. Stephanie, L. Loanardo, and C. Wijaya, Design and Financial Analysis of a Health Insurance Based on an SIH-type Epidemic Model, *Journal of Mathematics and Computer Science*, **38** (2025), 160–178.

Exploring stability and chaos in discrete dynamical systems

Syeda Jabeen*†1

[1]Birla Institute of Technology [Mesra] (BIT Mesra), Birla Institute of Technology, MesraRanchi, Jharkhand 835215, India
†E-mail: syed_sdj@yahoo.co.in

Abstract

This study explores the dynamics of discrete-time dynamical systems modeled by both linear and nonlinear difference equations. It examines how system states evolve over time in response to varying initial conditions and parameter values. Fundamental concepts such as equilibrium points, stability analysis, bifurcation phenomena, and periodic orbits are thoroughly examined using both analytical approaches and computational simulations. Special emphasis is placed on understanding the qualitative behavior of solutions and identifying conditions that lead to chaotic dynamics. The findings offer valuable insights into the underlying structure and predictability of discrete dynamical systems, with broad applications in fields such as ecology, economics, and engineering.

Keywords: Discrete, Time dynamical systems, Equilibrium points, Bifurcation phenomena, Periodic orbits

Extreme events in the network of FitzHugh–Nagumo neurons under the influence of poisson process

Nataliia Nikishina*†1 and Andrei Bukh[1]

[1] *Chernyshevsky Saratov National Research State University [Saratov] (SSU), 83 Astrakhanskaya street, city Saratov, Saratov region, country Russia, Russia*
† *E-mail: nnikishina7@gmail.com*

Abstract

Real networks of neurons are almost always located in conditions of external environmental influences, which are often extreme in nature. Extreme events are those that have an irregular intensity over time and, with an outbreak of activity, lead to a change in the normal functioning of a system. Regular effects in the form of pulse signals make possible to control the processes in neural systems [2]. The most common biologically relevant and simplest neuron model is a FitzHugh–Nagumo oscillator [1]. The network of this neuron model shows how the brain works, and also helps to simplify study the effects of various kinds of influences on the dynamics of neurons. So this kind of study is important and actual. A system of coupled neurons was used to study extreme events in the FitzHugh–Nagumo neural network.

We study the system with the excitability threshold value $|a| > 1$, when an individual FitzHugh–Nagumo oscillator demonstrates an excitable regime. Connections between neurons subject to periodic boundary conditions ($i + kN = i$ for any integer k). Poisson pulses were used as an external influence, where f is a frequency parameter and A is an amplitude of the signal. The Poisson pulses are characterized by an independent time between the moments of the pulses and have an exponential distribution.

The occurrence of extreme events is observed in the system with the probability less than 10 percent and can occur with different values of parameters under influence of the Poisson pulses.

This work was supported by the Russian Science Foundation (Project No. 23-72-10040, `https://rscf.ru/project/23-72-10040/`).

Keywords: FitzHugh–Nagumo neuron, Extreme events, Poisson pulses

References

[1] FitzHugh R., Mathematical Models of Threshold Phenomena in the Nerve Membrane, *The bulletin of mathematical biophysics*, **17(4)**(1955), 257-278.

[2] Tass P. A., Desynchronizing Double-pulse Phase Resetting and Application to Deep Brain Stimulation, *Biological Cybernetics*, **85(5)**(2001), 343-354.

Global bifurcations of limit cycles in continuous and discrete dynamical systems

Valery Gaiko*†1

1 *United Institute of Informatics Problems, National Academy of Sciences of Belarus, Surganova Str. 6, Minsk 220012, Belarus*
† *E-mail: valery.gaiko@gmail.com*

Abstract

Global bifurcations of limit cycles are studied in continuous and discrete dynamical systems. First, using new bifurcation and topological methods, we solve *Hilbert's Sixteenth Problem* on the maximum number of limit cycles and their distribution for the 2D Leslie-Gower population dynamics system, reduced quartic Topp system, and Euler-Lagrange-Liénard polynomial system. Then, applying a similar approach, we study 3D polynomial systems and complete the strange attractor bifurcation scenario for Lorenz-type systems connecting globally the Andronov, Shilnikov, homoclinic, period-doubling, period-halving and other bifurcations of limit cycles which is related to *Smale's Fourteenth Problem*. We discuss also how to apply our approach for studying global limit cycle bifurcations of multi-parameter discrete dynamical systems which model the population dynamics in biomedical and ecological systems.

Keywords: Dynamical system, Field rotation parameter, Bifurcation, Singular point, Limit cycle

References

[1] V. A. Gaiko, Global Bifurcation Theory and Hilbert's Sixteenth Problem, *Kluwer Academic Publishers*, 2003.

[2] V. A. Gaiko, C. Vuik, Global dynamics in the Leslie–Gower model with the Allee effect, *Int. J. Bifurcation Chaos*, **28** (2018), 1850151.

[3] V. A. Gaiko, H. W. Broer, A. E. Sterk, Global bifurcation analysis of Topp system, textitCyber. Phys., **8** (2019), 244–250.

[4] V. A. Gaiko, Limit cycles of multi-parameter polynomial dynamical systems, *J. Math. Sci.*, **260** (2022), 662–677.

[5] V. A. Gaiko, Global bifurcation analysis of the Lorenz system, *J. Nonlinear Sci. Appl.*, **7** (2014), 429–434.

Global dynamics and periodicity in a three-dimensional rational difference system

Hiba Zabat*†1, Nouressadat Touafek[1], Mehmet Gumus‡2 and Imane Dekkar[1]

[1] *Université Mohammed Seddik Benyahia [Jijel], University of Jijel (UMSBJ), Ouled Aissa, BP 98, 18000 Jijel, Algeria*
[2] *Zonguldak Bülent Ecevit University (BEU), Turquie, Turkey*
† *E-mail: zaabat.hiba@gmail.com*
‡ *E-mail: m.gumus@beun.edu.tr*

Abstract

In this presentation, we examine the global behavior of solutions to a nonlinear three-dimensional discrete dynamical system given by:

$$\begin{aligned} u_{n+1} &= a + v_{n-1}^{p}/(bv_{n}^{p} + cv_{n-1}^{p}) \\ v_{n+1} &= d + v_{n-1}^{q}/(ev_{n}^{q} + fv_{n-1}^{q}) \\ w_{n+1} &= \left[(\alpha u_{n}^{s} + \beta u_{n-1}^{s})/(\gamma u_{n}^{s} + \lambda u_{n-1}^{s})\right] u_{n}^{t} u_{n-1}^{t}, \quad n \in \mathbb{N}, \end{aligned}$$

where $p, q, r \in \mathbb{N}$, $s, t \in \mathbb{N}_0$. The parameters $a, b, c, d, e, f, \alpha, \beta, \gamma, \lambda$ and the initial conditions u_0, u_{-1}, v_0, v_{-1}, w_0, w_{-1} are positive real numbers.

We present sufficient conditions for the local and global stability of the system's unique equilibrium, along with criteria for the existence of periodic solutions. Numerical examples will be presented to support and illustrate the theoretical results.

Keywords: Global stability, Local stability, Periodicity, Equilibrium point

References

[1] Y. Akrour, M. B. Mesmouli, D. T. Tollu and N. Touafek, On the Solutions of a System of Max-Type Difference Equations, *ASETMJ*, **14(4)** (2021), 159–187.

[2] E. Beso, S. Kalabusic, N. Mujic, E. Pilav, Boundedness of Solutions and Stability of Certain Second-Order Difference Equation With Quadratic Term, *Adv. Difference Equ.*, **19**(2020), 22 p.

[3] I. Dekkar, N. Touafek, Q. Din, On the Global Dynamics of a Rational Difference Equation With Periodic Coefficients, *J. Appl. Math. Comput.*, **60** (2019), 567–588.

[4] I. Dekkar, N. Touafek, Global Stability of Some Nonlinear Higher-Order Systems of Difference Equations, *Dyn. Contin. Discrete. Impuls. Syst. Ser. A Math. Anal.*, **27(2)**(2020), 131–152.

[5] R. Devault, V. L. Kocic and D. Stutson, Global Behavior of Solutions of the Nonlinear Difference Equation $x_{n+1} = p_n + \frac{x_{n-1}}{x_n}$, *J. Difference Equ. Appl.*, **11(8)** (2005), 707–719.

[6] S. Elaydi, An Introduction to Difference Equations, *Undergraduate Texts in Mathematics*, Springer, 2005.

Influence of the Allee effect on prey-predator population dynamics: A mathematical and ecological perspective

Kalyan Das*†[1,2]

[1] *Department of Interdisciplinary Sciences, National Institute of Food Technology Entrepreneurship and Management, Plot No.97, Sector 56, HSIIDC Industrial Estate, Kundli, Sonepat, Haryana, 131028, India*
[2] *National Institute of Food Technology Entrepreneurship and Management (NIFTEM-K), Plot No.97, Sector 56, HSIIDC Industrial Estate, Kundli, Sonepat, Haryana, 131028, India*
† *E-mail: daskalyan27@gmail.com*

Abstract

This study explores the impacts of the Allee effect on prey-predator population dynamics through a mathematical modeling approach. The Allee effect, characterized by a positive correlation between individual fitness and population density at low population sizes, introduces significant complexity into ecological interactions. By incorporating the Allee effect into a classical prey-predator model, we investigate how this phenomenon influences equilibrium points, stability conditions, and population trajectories. Analytical and numerical methods are employed to identify critical thresholds where population extinction, coexistence, or oscillatory behavior emerges. The results highlight that the Allee effect can alter stability regions and may trigger bifurcations, leading to abrupt shifts in population dynamics. This research underscores the importance of considering the Allee effect in ecological modeling to better predict species survival and inform conservation strategies. Also our studies is extended these findings by incorporating discrete delay as additional ecological factors and impact of Allee effect for the discrete version of the same model.

Keywords: Allee effect, Prey, Predator dynamics, Stability analysis, Time delay, Bifurcation, Biological control

References

[1] W. C. Allee, Animal Aggregations, University of Chicago Press, Chicago (1931).

[2] M. Wang, M. Kot, Speeds of Invasion in a Model With Strong or Weak Allee Effects, *Math. Biosci.*, **171** (2001), 83–97.

[3] Y. Cai, C. Zhao, W. Wang, and J. Wang, Dynamics of a Leslie-Gower Predator-Prey Model With Additive Allee Effect, *Appl. Math. Model.*, **39(7)** (2015), 2092–2106.

[4] B. D. Hassard, N. D. Kazarinoff, Y. H. Wan, Theory and Application of Hopf Bifurcation, Cambridge University Press, Cambridge (1981).

On one of Birkhoff's theorems for backward limit points

Veronika Rýžová*†1

[1] *Silesian University in Opava, Univerzitní nám. 1934/3a 733 40 Karviná Czech Republic, Czech Republic*
† *E-mail: veronika.ryzova@math.slu.cz*

Abstract

In 1927 George Birkhoff in his book Dynamical Systems presented a theorem that describes the behaviour of trajectories outside of a set of non-wandering points on an arbitrary compacta. Much later in 1960s Sharkovsky followed up on Birkhoff's work and published even stronger result, this time focusing on the set of omega limit points for interval maps. We formulate similar statement for a neighbourhood of a set of different types of backward limit points for maps of the interval.

References

[1] G. D. Birkhoff, Dynamical Systems, *Amer. Math. Soc.* (1927).

[2] M. Forys-Krawiec, J. Hantakova, P. Oprocha, On the Structure of α-Limit Sets of Backward Trajectories for Graph Maps, *Discrete Contin. Dyn. Syst.*, **42** (January 2021).

[3] O. M. Sharkovsky, Attractors of Trajectories and Their Basins (Russian), *Naukova Dumka, Kyiv* (2013).

Polynomial-like iterative equation on Riesz spaces

Chaitanya Gopalakrishna*†1

[1] *Indian Statistical Institute (ISI) - R. V. College Post, 8th Mile Mysore Road, Bengaluru 560 059, India*
† *E-mail: cberbalaje@gmail.com*

Abstract

In this talk we investigate the polynomial-like iterative equation on Riesz spaces. Since a Riesz space does not need to have a metric space structure, neither the Schauder fixed point theorem nor the Banach fixed point theorem are applicable. Using the Knaster–Tarski fixed point theorem, we first discuss the existence and uniqueness of order-preserving solutions on convex complete sublattices of Riesz spaces. Then, restricting to $\mathbb{R}$ and $\mathbb{R}^n$, special cases of Riesz space, we discuss semi-continuous solutions and integrable solutions, respectively. Finally, we present more special cases of Riesz space in which solutions to the iterative equation can be discussed. The talk is based on a joint work with Weinian Zhang [1].

Keywords: Functional equation, Iteration, Complete lattice, Riesz space, Order preserving map

References

[1] C. Gopalakrishna, W. Zhang, Polynomial-Like Iterative Equation on Riesz Spaces, *Positivity*, **28(55)** (2024), 1–17.

Pricing real estate index Asian options using a nonstandard finite difference method

Mbakisi Dube[1] and Kailash Chandra Patidar[*†1]

[1] *University of the Western Cape (UWC), Robert Sobukwe Road, Bellville, 7535, Republic of South Africa, South Africa*
[†] *E-mail: kpatidar@uwc.ac.za*

Abstract

In this work, we consider a real estate index based Asian option pricing problem. The underlying, which is a real estate index, possesses serial correlation and predictability. As a result it respectively has positive and negative autocorrelation in the short and long run. A model that captures such characteristics is a mean reverting stochastic process. We establish the dynamics of the mean reversion model for the real estate index under an equivalent martingale measure ensuring the absence of arbitrage and a fair pricing framework. We use the First Fundamental Theorem of Asset Pricing along with the Feynman-Kac Theorem to formulate a partial differential equation for the price of a fixed strike real estate index Asian call option. Resulting model is simulated using a nonstandard finite difference scheme that we develop and analyze in this paper. Competitive numerical results will be presented at the conference.

Keywords: Computational finance, Option pricing, Nonstandard finite difference methods

The existence of hopf bifurcation for a delayed Holling-Tanner type predator-prey model

Adalet Zeynep Esirgen*†1

[1] *Yildiz Technical University (YTU), Barbaros Bulvarı 34349 Yıldız-İstanbul, Turkey*
† *E-mail: zeynep.esirgen@gmail.com*

Abstract

In this paper, a Holling-Tanner type predator-prey model with discrete time delay is investigated where the functional response of predator dynamics is ratio dependent. We first consider the local stability of the positive equilibrium and the existence of Hopf bifurcations. Then, by choosing the delay time τ as the bifurcation parameter, we show that Hopf bifurcation occurs as the delay time τ passes some critical values. Moreover, we perform the numerical simulations to justify the existence result.

Keywords: Predator, Prey system, Discrete delay, Hopf bifurcation, Stability

Tree harvesting problem and forest valuation under real constraints

Tomas Tichy*†1

[1] *Technical University Ostrava (VSB-TUO), Czech Republic*
† *E-mail: tomas.tichy@vsb.cz*

Abstract

Managers' strategic decisions under uncertainty and changing market conditions play a significant role in the forestry sector as well. This paper addresses the optimal tree harvesting problem under logging restrictions and mean-reverting lumber prices. Using contingent claims analysis, we develop a PDE model that values forest land as a real option, akin to a call option with European and American exercise rights. The model incorporates a stochastic mean-reverting nature of lumber prices and applies the discontinuous Galerkin method for numerical solutions. Special attention is also paid to the bare land valuation that allows us to formulate the forest valuation procedure as a multi-rotation problem while maintaining one spatial dimension. Our study considers a given interval of the prices of market risk and highlights several key findings: (1) Flexible harvesting policies significantly impact forest stand values, with earlier harvesting policies yielding higher values; (2) The critical harvesting prices decrease with the age of the forest stand, indicating optimal harvesting times; (3) Traditional Faustmann's rule underestimates forest stand values compared to our real options approach. These findings underscore the importance of incorporating market price risk, flexibility, and multiple rotations in forest valuation models to make informed and optimal harvesting decisions. Our case study, set in a real-world framework, demonstrates the financial and economic implications of different harvesting strategies, providing a comprehensive framework for valuing forest investments under uncertainty.

Keywords: Forest valuation, Mean reversion, Optimal harvesting, Logging restrictions, Real option, Discontinuous Galerkin method

On limit sets and fractional iterates of Brouwer homeomorphisms

Zbigniew Leśniak*†1

1 *University of the National Education Commission, Krakow*
† *E-mail: zbigniew.lesniak@uken.krakow.pl*

Abstract

We present properties of the set of irregular points of a Brouwer homeomorphism f for which there exists a foliation of the plane whose leaves are invariant lines of f. For such homeomorphisms, we give the form of the limit set of a strongly irregular point. Based on this result, we present a method for constructing continuous, orientation-preserving fractional iterates of f. These iterates are constructed recursively on a family of maximal parallelizable regions of f. To ensure continuity of the fractional iterates, we apply a matching property along the boundaries of the regions.

References

[1] A. Gasull, J. Groisman, F. Manosas, Regularization of Brouwer Translation Theorem, *Qual. Theory Dyn. Syst.* **21**:135 (2022), 9 pp.

[2] F. Le Roux, A. G. O'Farrell, M. Roginskaya, I. Short, Flowability of plane homeomorphisms, *Ann. Inst. Fourier Grenoble* **62** (2012), 619–639.

[3] Z. Lesniak, On families of invariant lines of a Brouwer homeomorphism, *J. Difference Equ. Appl.* **25** (2019), 1363–1371.

[4] Z. Lesniak, On limit sets of a Brouwer homeomorphism generating foliations of the plane, *J. Difference Equ. Appl.*, 1–13, `https://doi.org/10.1080/10236198.2025.2509779`