A Tribute to Professor Stephen Smale on His 95th Birthday

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In this article, I will share a personal perspective on my highly revered icon Stephen Smale.

Steve became a Professor Emeritus at Berkeley in 1995 and subsequently joined the City University of Hong Kong (CityUHK) as a Distinguished University Professor, a position he held until 2001. I joined CityUHK towards the end of 1999 and became his colleague and friend.

In 2002, Steve left CityUHK to work as Professor at the Toyota Technological Institute in Chicago till 2009. He then returned to CityUHK and remained there until 2016. After retiring as Professor Emeritus with the title Honorary Professor of CityUHK in 2016, he settled back in his home at Berkeley.

During his time at CityUHK, we met frequently and developed a somewhat close friendship. I stopped by his office for a brief chat almost every weekday after grabbing coffee from the college room just across the hallway. We also went hiking together several times in Hong Kong, usually along with some other colleagues and students. In April 2010, I arranged a special trip to Beijing with him alone, during which we visited the Chinese Academy of Sciences, Peking University, and Tsinghua University where he gave lectures and seminars.

Through casual conversations, I learned more about Steve both personally and professionally. However, I must admit that Steve is not particularly talkative. Therefore, the brief overview of his career and achievements given in this informal short article is mainly based on my reading of numerous references, and I acknowledge that it may not be entirely complete and accurate.

Although my knowledge is limited, I am truly honored to write this brief tribute celebrating Steve's 95th birthday on July 15, 2025.

Part I.

Steve's Career Path and Main Achievements

Steve was born on July 15, 1930 in Flint, Michigan, USA. He earned his B.S. degree in 1952, M.S. degree in 1953, and Ph.D. in 1957——all in mathematics from the University of Michigan. His doctoral thesis, titled “Regular curves on Riemannian manifolds,” extended certain analytical results of Hassler Whitney (1907–1989), generalizing from regular closed curves on the plane to those on n-dimensional manifolds.

Steve’s academic career began as an Instructor at the University of Chicago from 1956 to 1958. In 1958, he proved his famous theorem demonstrating that a 2-sphere in 3-dimensional space can be turned inside out through immersions——a groundbreaking result in topology. During 1958–1959, he worked at the Institute for Advanced Study in Princeton, where he developed significant Morse inequalities pertinent to dynamical systems.

In 1959–1960, Steve was awarded the National Science Foundation Postdoctoral Fellowship. Using this support, he visited the Instituto de Mathematica Pura e Aplicada in Rio de Janeiro, Brazil, from January to June in 1960. During this period, he established the rigorous mathematical theory of horseshoe maps in chaos theory. Moreover, he made a landmark contribution by proving the Poincaré conjecture for all dimensions $n \geq 5$, a world-stunning result, which was published in 1961. The conjecture of Jules Henri Poincaré (1854–1912) was one of the most famous challenging mathematical problems of the 20th century. Steve’s foundational contributions earned him the Fields Medal in 1966, awarded for proving the generalized Poincaré conjecture in dimensions $n \geq 5$: "Every closed, n-dimensional manifold homotopy-equivalent to the n-sphere is homeomorphic to it."

Additionally, in 1962, Steve generalized the h-cobordism theorem to higher-dimensional manifolds. In 1965, he extended the Morse-Sard theorem on critical values of smooth functions to a broad class of nonlinear maps in infinite-dimensional Banach spaces, significantly impacting topology and dynamical systems.

In the late 1960s, building on his earlier successes, Steve shifted his focus toward more application-oriented research. Over the subsequent years, he extended his mathematical theories to a wide range of scientific fields, including economics, celestial mechanics, circuit electronics, biology, immunology, and multi-agent flocking models.

In recognition of his groundbreaking works, Steve was awarded the Wolf Prize in Mathematics in 2007, with the citation highlighting his influential contributions to differential topology, dynamical systems, mathematical economics, and other areas of mathematics.

Throughout his distinguished career, Steve held positions at several prestigious institutions. He worked as an Associate Professor at the University of California, Berkeley (1960–1961) and as a Professor at Columbia University (1961–1964). He then returned to UC Berkeley as a Full Professor, a role he held until his retirement in 1995. To that end, as mentioned earlier, he joined CityUHK as a Distinguished University Professor. In 2002, he left CityUHK to become a Professor at the Toyota Technological Institute in Chicago, where he worked until 2009. He subsequently rejoined CityUHK as a Distinguished University Professor. Steve retired completely in 2016 at the age of 86, retaining the title of Honorary Professor at CityUHK, and settled back in his home at Berkeley.

Part II.
Steve's Main Contributions in Mathematics

It is virtually impossible for me, especially within the narrow scope of this short article, to fully capture and describe Steve's foundational contributions to mathematics. Nonetheless, I will endeavor to provide a brief overview of what I believe are his most significant achievements, based on my limited personal knowledge.

Differential Topology

In 1958, Steve learned the work of Aleksandr A. Andronov (1901–1952) and Lev S. Pontryagin (1908–1988) on the structural stability of dynamical systems and tried to apply topological methods to study those problems. That research effort led to the Morse–Smale systems theory, revealing a class of structurally stable smooth dynamical systems whose non-wandering set consists of finitely many hyperbolic equilibrium points and hyperbolic periodic orbits, satisfying a transversality condition on the stable and unstable manifolds.

In 1966, Steve received a Fields Medal in the International Congress held in Moscow, USSR. One of his significant contribution was his work on the generalized Poincaré conjecture. The general version of the conjecture is in the higher-dimensional space, which claims that any closed n-dimensional manifold that is homotopy-equivalent to the n-sphere must be the n-sphere itself. When $n = 3$, this is the classical Poincaré conjecture. Steve proved the higher-dimensional version for $n \geq 5$. Steve solved the problem based on the Morse theory, for which he had made very substantial contributions. Morse theory enables one to analyze the topology of a manifold by studying differentiable functions on that manifold. Later in 1982, Michael H. Freedman (1951–) proved the generalized Poincaré conjecture for $n = 4$ and, due to that result, received a Fields Medal in 1986. The original conjecture for $n = 3$ was finally settled by Grigori Y. Perelman (1966–), who was offered the 2006 Fields Medal but did not claim it.

In addition to the Fields Medal, Steve was awarded the 1966 Veblen Prize for Geometry by the American Mathematical Society, "for his contributions to various aspects of differential topology".

Dynamical Systems and Chaos Theory

Steve made a significant impact on the field of dynamical systems theory in the 1960s.

In the late 1950s and early 1960s, inspired by his study of the collected works of George D. Birkhoff (1884–1944), Steve discovered a crucial connection between Poincaré's homoclinic points and the fundamental properties of dynamical systems. Homoclinic points provided him with examples of structurally stable systems containing infinitely many periodic points. In these systems, minimal sets are homeomorphic to Cantor sets, which could be analyzed using his emerging hyperbolic systems theory from the viewpoint of stable and unstable manifolds.

After arriving in Brazil as an NSF Postdoc Fellow in the early 1960, Steve received a letter from Norman Levinson (1912–1975), informing him that there exist structurally stable systems that are not Morse–Smale systems. This insight prompted Steve to develop the horseshoe map, which describes a strange attractor within a dynamical system——later recognized as a hallmark of chaotic systems. Since then, the horseshoe map theory has become a powerful tool for analyzing chaos in dynamical systems. It provides a means to prove the presence of chaos, characterized by the existence of infinitely many periodic orbits, including orbits of arbitrarily long periods, as well as other complex behaviors especially the sensitivity to initial conditions.

In 1967, Steve published the seminal article "Differentiable dynamical systems," which laid a topological foundation for rigorous mathematical approaches to analyzing chaos.

In his 1998 article "Chaos: Finding a horseshoe on the beaches of Rio," published in Mathematical Intelligencer, Steve recalled: "I was lucky to find myself in Rio at the confluence of three different historical traditions in the subject of dynamics (called ordinary differential equations at the time)." These "three historical traditions" refer to: the work of Andronov and Pontryagin within the Soviet Gorki school on nonlinear dynamics; the contributions of Norman Levinson, Mary L. Cartwright (1900–1998) and John L. Littlewood (1885–1977) on the van der Pol oscillator; the foundational work of Poincaré and Birkhoff on the qualitative theory of ordinary differential equations.

Specifically, Steve proved that diffeomorphisms with transverse homoclinic points possess nearby hyperbolic invariant sets on which the dynamics are conjugate to a shift on a finite alphabet of symbols. He also completed Poincaré's work by establishing a connection between ordinary differential equations and deterministic maps with probabilistic Markov processes, demonstrating that their orbits are indistinguishable. This work culminated in the proof of the Smale-Birkhoff homoclinic theorem.

In this area of research, as Editor-in-Chief of the International Journal of Bifurcation and Chaos, I had the honor of inviting Indika Rajapakse (1980-) and Steve to publish their paper “The pitchfork bifurcation” in the journal in 2017. In this work, they present a new theory of the pitchfork bifurcation that removes the traditional requirements of symmetry and the third derivative, advancing the understanding of bifurcation phenomena in dynamical systems.

Part III.

Steve's Main Contributions in Mathematical Foundation of Scientific Applications

Starting from the late 1960s, after achieving significant success, Steve began to shift his research focus toward various applied scientific fields. Specifically, he explored how his mathematical theories could lay the groundwork in such areas as economics, celestial mechanics, circuit electronics, biology, immunology, and multi-agent flocking models. He modeled physical processes——including for instance the n-body problem and electric circuit dynamics——using dynamical systems theory, often employing topological methods, to deepen the understanding across these diverse disciplines.

Mathematical Economics

In 1968, Steve met economist Gerard Debreu (1921–2004) at UC Berkeley, where they collaborated on exploring mathematical theories in economics. Throughout the 1970s, Steve focused on general economic equilibrium, applying his topological and dynamical systems approaches. He published a series of papers on the dynamics of price adjustment, gaining a deep understanding of the nature and structure of economic equilibria. Building on this foundation, Steve also developed constructive methods for their computation.

In 1983, Debreu was awarded the Nobel Prize in Economics, and he specifically credited the Sard theorem——introduced to him by Steve——as a key factor in the development of his main economics theory.

Published in the 1980s, the three-volume Handbook of Mathematical Economics, compiled by Nobel laureates and leading economists, include a chapter authored by Steve titled "Large-scale analysis and economics," highlighting his influential contributions to the field.

Computation Theory

In an effort to unify the fields of theoretical computer science and numerical analysis, Steve collaborated with Lenore C. Blum (1942–) and Michael I. Shub (1943–) on developing a computational model that integrates aspects of both the Turing machine approach and numerical analysis methods. Together, they introduced the "Blum-Shub-Smale machine", an alternative framework to the classical Turing model of computation, designed to analyze the computability of functions within theoretical computer science.

The first comprehensive textbook on this important subject is Complexity and Real Computation, published in 1998 and coauthored by Lenore Blum, Steve Smale, Mike Shub, and Juan Felipe Cucker (1958–). In this work, they demonstrate that classical complexity theory, based on the Turing model, is insufficient for addressing many problems and algorithms encountered in modern scientific computing. The book then develops a new complexity theory tailored to these applications. In the Introduction, it states that “The point of view of this book is that the Turing model (we call it ‘classical’) with its dependence on 0’s and 1’s, is fundamentally inadequate for giving such a foundation for modern scientific computation, where most of the algorithms——with origins in Newton, Euler, Gauss, et al. ——are real number algorithms.”

In this area of research, Steve and his collaborators established rigorous mathematical foundations for practical algorithms, including Newton’s method and the simplex method for linear programming, among others. They also developed techniques and tools for analyzing these algorithms’ performance and properties.

In the summer of 1995, Steve organized a month-long conference in Park City, Utah, focused on the mathematical theory of computation. The goal was “to strengthen the unity of mathematics and numerical analysis, and to narrow the gap between pure and applied mathematics.” Following this, Steve founded the Society for the Foundations of Computational Mathematics and established the journal Foundations of Computational Mathematics. Since 2011, the Society created the Stephen Smale Prize to honor outstanding contributions by young mathematicians in the broad field of computational theory, awarding the prize once every three years.

Learning Theory

In the early 2000s, Steve collaborated with Juan Felipe Cucker and Ding-Xuan Zhou (1967–) to formalize learning theory within a rigorous mathematical framework. This theory bridges approximation theory and data-driven statistical learning, offering insights into numerous “learning” activities——such as language acquisition by children, the emergence of languages in early human cultures, sensor-based design in manufacturing engineering, and pattern recognition in image analysis. Its applications span a wide range of disciplines, including cognitive psychology, animal behavior, economic decision-making, and all branches of engineering and computer science——particularly in understanding how the brain functions and human thought processes.

Steve applied his structural stability theory from dynamical systems to study how learning processes can converge to stable solutions and to analyze the behavior of iterative learning algorithms. He and his collaborators investigated the structures of neural networks and examined the capacity of specific function classes to approximate arbitrary continuous functions. This work shed light on the theoretical limits of various learning models and their ability to generalize. They analyzed gradient descent methods within the context of learning, demonstrating convergence properties and stability in infinite-dimensional spaces such as reproducing kernel Hilbert spaces. Additionally, they re-examined the Shannon sampling theorem and explored the statistical nature of sampling data in function reconstruction and approximation for learning analysis.

Furthermore, Steve collaborated with Tomaso Armando Poggio (1947–) from a neural science perspective to develop rigorous mathematical methods and models for machine learning, which have significantly influenced the development of modern deep learning theory.

In earlier 2000, Steve worked with Yuan Yao (1973–) on establishing a mathematical foundation for dynamic theory of learning, which motivates extensive studies on modern machine learning theory and methods including neural networks.

Around 2010, Steve also worked on topological data analysis utilizing concepts from Hodge theory, particularly through the lens of discrete Hodge theory, to analyze data represented as point clouds or simplicial complexes. Actually, as earlier as in the 1980s, Steve already made significant contributions to the understanding of the simplex method in linear programming, particularly in analyzing its average complexity and performance.

Part IV.

Steve's Achievements, Awards and Honors

Steve was awarded the 1966 Fields Medal "for his pioneering work in topology and differential geometry, especially his proof of the generalized Poincaré conjecture for higher dimensions and his development of the theory of dynamical systems."

In 2007, Steve received the Wolf Prize "for his groundbreaking contributions that have played a fundamental role in shaping differential topology, dynamical systems, mathematical economics, and other areas of mathematics."

In 1996, Steve was honored with the US National Medal of Science in recognition of his "four decades of pioneering work on fundamental research questions that have led to major advances in both pure and applied mathematics."

In addition, Steve was conferred honorary degrees by University of Warwick (1974), Queens University, Kingston, Ontario (1987), the University of Michigan (1996), Université Pierre et Marie Curie, Paris (1997), City University of Hong Kong (1997), Rostov State University (1999), and University of Genoa (2004).

Steve was elected honorary member of the Instituto de Matematica Pura e Aplicada, Rio de Janeiro (1990), Trinity Mathematical Society, Dublin (1991), Moscow Mathematical Society (1997) and the London Mathematical Society (1998).

In addition to the above honors and prizes, Steve was awarded the Veblen Prize by the American Mathematical Society (1966), the Chauvenet Prize by the Mathematical Association of America (1988), the von Neumann Award by the Society for Industrial and Applied Mathematics (1989), and the Juergen Moser Prize by the same Society (2005).

More significantly, Steve was elected foreign member of the Brazilian Academy of Sciences (1964), member of the American Academy of Arts and Sciences (1967), and member of the US National Academy of Sciences (1970).

Last but not least, in 1973, the Crimean Astrophysical Observatory and Institute of Applied Astronomy of Russian Academy of Science named a newly discovered minor planet "Smale Planet".

Part V.
Steve's Life Stories

Steve's life stories are rich, compelling and inspiring. I would like to highlight just a few of his professional as well as personal endeavors.

Social and Political Activities

In the 1960s, Steve was deeply involved in political and social issues. He actively supported the civil rights movement and opposed the Vietnam War, participating in protests and demonstrations advocating for racial equality and global peace during this turbulent decade in the US.

At the 1966 International Congress of Mathematicians held in Moscow, USSR, Steve was awarded the Fields Medal. During his speech, he notably criticized both the United States and the Soviet Union regarding challenges related to peace and democratization.

In 1998, Steve published a well-known article titled "Chaos: Finding a horseshoe on the beaches of Rio," in which he reflected on his experiences with frustrations and conflicts involving the US National Science Foundation (NSF). The article recounts how, during his 1960 NSF Postdoctoral Fellowship——initially criticized by the NSF——he conceived the horseshoe map, a fundamental concept in chaos theory, inspired by thought-provoking research conducted on the Rio beaches during his postdoctoral studies in Brazil.

Some Personal Life Stories

Steve was a frequent user of the library, especially during his early research days. However, I was quite surprised to find that his office bookshelves at CityUHK contained no books, only manuscripts. I chose not to ask him directly about this, because I remembered Alexander Grothendieck (1928–2014), who shared the Fields Medal with Steve in 1966. When asked about his office at IHÉS, France, Grothendieck replied: "We do not read books. We write books."

Steve is a passionate mountain hiker and has been an accomplished sailor for many years. During my time in Hong Kong, I had the great pleasure of hiking with him several times, often accompanied by colleagues and students. He was always the energetic leader, guiding the group along mountain trails.

Most interestingly, Steve assembled one of the finest collections of crystals in the world. He produced excellent photographs of his specimens and displayed his beautiful and valuable stones in several exhibitions at CityUHK. Today, many of his mineral specimens can be found in his book The Smale Collection: Beauty in Natural Crystals.

Aspiring Learning Experience

In studying and researching mathematics, I find inspiration in a quote from Steve: “At least in my own case, understanding mathematics doesn't come from reading or even listening. It comes from rethinking what I see or hear. I must redo the mathematics in the context of my particular background. And that background consists of many threads, some strong, some weak. My background is stronger in geometric analysis, but following a sequence of formulae gives me trouble. I tend to be slower than most mathematicians to understand an argument. The mathematical literature is useful in that it provides clues, and one can often use these clues to put together a cogent picture. When I have reorganized the mathematics in my own terms, then I feel an understanding, not before.”

18 Mathematical Problems for the 21st Century

In 1998, Steve compiled a list of eighteen “Mathematical problems for the next century” on the following topics:

1. The Riemann hypothesis;
2. The Poincaré conjecture;
3. Does P = NP?
4. Integer zeros of a polynomial;
5. Height bounds for Diophantine curves;
6. Finiteness of the number of relative equilibria in celestial mechanics;
7. Distribution of points on the 2-sphere;
8. Introduction of dynamics into economic theory;
9. The linear programming problem;
10. The closing lemma;
11. Is one-dimensional dynamics generally hyperbolic?
12. Centralizers of diffeomorphisms;
13. Hilbert’s 16th problem;
14. Lorentz attractor;
15. Navier-Stokes equations;
16. The Jacobian conjecture;
17. Solving polynomial equations;
18. Limits of intelligence.

Some of these challenging mathematical problems have been solved lately.

Epilogue

To conclude this article, I would like to mention that Steve's former PhD student Michael (Xiaohua) Xuan (1963-), from UC Berkeley, founded UniDT(Shanghai) Co., Ltd.. He and UniDT initiated *The Smale Institute for Mathematics and Computation*, which focuses on the foundational theory of computation and algorithms. Steve serves as the Chairman Emeritus of the Institute, and I am honored to be a member of it.

It is a privilege for me to acknowledge, in this short article, a remarkable individual whose life and achievements have already made a lasting impact. I hold immense respect and admiration for Steve, whose wisdom and kindness serve as an inspiration to me and to all who know him.

May July 15, 2025 be filled with joy and happiness for Steve on this special day.

Steve and Ron (taken from Steve's office at CityUHK on December 11, 2015)